УДК 621.541.1
DOI: http://dx.doi.org/10.20535/2521-1943.2017.81.120008

Numerical model of the pneumatic diaphragm actuator

Aleksandr Kovalenko

Cherkasy State Technological University, Cherkasy, Ukraine

Received: 20 October 2017 / Accepted: 27 November 2017

Abstract. *The paper deals with the development of the numerical model of a pneumatic diaphragm actuator to be used in numerical modeling and investigation of such systems as automobile suspension and braking systems, machine tool devices, pressure control valves, etc.*
The key point of the investigation was development of the diaphragm model and its key parameter - effective area. It has been found that the accurate enough diaphragm effective area can be determined as a function of readily available and easily measurable parameters such as the rigid center displacement, pressures in the chambers, diaphragm dimensions, and its material physical properties.
The accuracy of the model was assessed by testing the diaphragm itself and in simulation of various technical systems with pneumatic diaphragm actuators.

Keywords: diaphragm; actuator; numerical model; effective area; non-linear diaphragm rigidity.

Introduction

Diaphragm actuators are widely used in engineering in various devices: automobile suspension and braking systems, machine tool devices, equipment actuators with heavy working conditions, pressure control valves, etc. The reasons are as follows – diaphragm actuators, in comparison with, for example, cylinders, have such advantages as low laboriousness in manufacturing, high tightness of the working chamber, low frictional force, no need for high-quality air no need for quality air preparation, low operating costs.

However, diaphragm actuators also have some disadvantages. They are low stroke, low diaphragm durability, and, one of the most significant disadvantage, the nonlinear law of the developed force, which can be determined as a function of the rigid center displacement, pressures in the chambers, diaphragm dimensions, and its material physical properties. In turn, the nonlinear low of the force is determined by the nonlinear dependence of the effective area and the elastic properties of the diaphragm along the length of the rigid center displacement.

To solve this problem, in some cases, where it is possible, diaphragm actuators with a small working stroke (1/4 - 1/3 maximum) are used, within which the nonlinearity of the effective area and the elasticity of the membrane affect insignificantly.

In other cases, there is both the need to determine the force developed by the diaphragm actuator, and the need to design a diaphragm actuator that develops a certain predetermined force.

Therefore, for the qualitative design of membrane drives, it is necessary to develop an adequate mathematical model (or models, taking into account the variety of applications and operating conditions) adapted to the use of numerical methods of solution as part of modern CAE systems.

The purpose of this study

The purpose of this study is the development of a numerical mathematical model of the pneumatic diaphragm chamber, which is the base part of pneumatic diaphragm actuators of various purposes, in which the effective area of the diaphragm is defined as a function of relatively easily and unambiguously measured physical quantities - the rigid center displacement, pressures in chamber, diaphragm dimensions, and its material physical properties (diaphragm and rigid center diameters, the presence and length of the goffer, the thickness and the modulus of elasticity of the diaphragm, etc.).

✉ Aleksandr Kovalenko
a.kovalenko@2upost.com

Under the development of a numerical model, the article refers to the adaptation of analytical dependencies which describes the pneumatic diaphragm chamber to the use of numerical methods of solution as part of modern CAE systems, checking the adequacy of the model developed by comparing the simulation results with experimental data, and running test examples.

Study

The design and calculation schemes of a typical diaphragm actuator are presented in Fig. 1. The mathematical model of the diaphragm chamber in the general combines the equations of filling and emptying the variable volume pneumatic diaphragm chamber, the equation for the thermal processes - changing the air temperature in the chamber caused by changing the air pressure in the chamber, and chamber volume as well as by heat exchange with the environment.

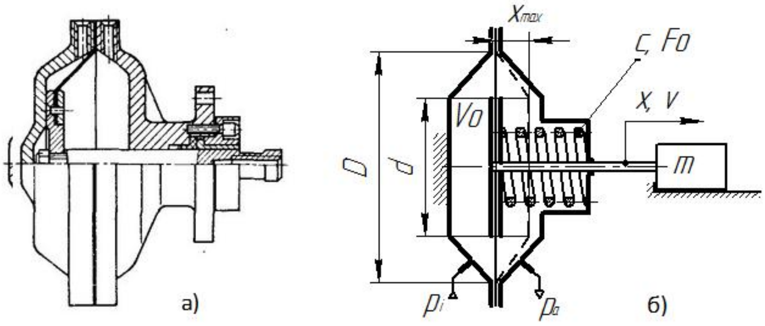

Fig. 1 Design (a) and calculation (b) schemes of a typical diaphragm actuator

The system of equations has the form:

$$\begin{cases} G = f_1(p,V,T) = G_V + G_p + G_T; \\ q = f_2(p,V,T) = q_U + q_L + q_T, \end{cases} \tag{1}$$

where G - air mass flow which enters into the chamber; q - heat flow carried by air; G_V, G_p, G_T - air mass flows, which spend for changing of the volume, pressure and temperature of the air in the chamber; $q_U + q_L + q_T$ - heat flows, describing, respectively, a change in the internal energy of the air in the chamber, the work performed by air or over it and on heat exchange with the environment. These components of the system of equations (1) are determined from the equation of state of the gas and the first law of thermodynamics and have the form:

$$G_V = \frac{A_e}{R \cdot T} \cdot p \cdot \frac{dx}{dt}; \qquad G_p = \frac{V_0 + A_e \cdot x}{R \cdot T} \cdot \frac{dp}{dt}; \qquad G_T = \frac{V_0 + A_e \cdot x}{R \cdot T^2} \cdot p \cdot \frac{dT}{dt};$$

$$q_U = \frac{c_V}{R} \cdot \left(p \cdot A_e \cdot \frac{dx}{dt} + (V_0 + A_e \cdot x) \cdot \frac{dp}{dt} \right); \qquad q_L = p \cdot A_e \cdot \frac{dx}{dt}; \qquad q_T = \alpha \cdot A_{HT} \cdot (T - T_W),$$

where A_e is the effective area of the diaphragm; V_0 is initial volume of the diaphragm chamber; R is the gas constant; c_V is the gas heat capacity at constant volume; α is the heat transfer coefficient; p is the gas absolute pressure in the chamber; T is the gas absolute temperature in the chamber; x is the diaphragm rigid center displacement; A_{HT}, T_W is the area and absolute temperature of the surface through which heat transfer takes place.

To create a mathematical model of a diaphragm actuator, the system of equations (1) must be supplemented by the equation of the balance of forces - the equation of motion of the rigid center and the masses associated with it under the action of variable pressure in the diaphragm chamber and forces acting on the membrane (springs, friction, inertia, loads).

Such “mechanicals” blocks as a rule widely represented in the modern CAE systems and they can be used for modeling. Various modifications of the diaphragm actuator models can be obtain by using different models of such elements - to determine the friction forces, stops, heat exchange, gas properties, etc.

So, in this paper, we consider the choice of the dependence for determining the effective area of the diaphragm only.

There are several mathematical relationships for determining the force developed by the diaphragm actuator [1, 2, 4, 5, 6, 7, 8, 9]. All of them are based on the determination of the effective area of the diaphragm - the fictitious area, whose product at the pressure drop across the diaphragm is equal to the force developed by the actuator.

1. Simplified dependence, according to which the diaphragm effective area assumed to be constant and depends only on the diameters of the diaphragm and rigid center, (Fig. 1). The dependence has the form [6]:

$$A_e = \frac{\pi \cdot D^2}{12} \cdot \left(\beta^2 + \beta + 1\right) \tag{2}$$

where $\beta = \frac{d}{D}$.

Due to significant errors, the range of application for this relationship is limited approximately by $1/4 - 1/3$ part of the full possible stroke of the rod.

2. Dependence, known as the Liktan formula [1]. A semi-empirical formula relating the diaphragm effective area to the rigid center displacement, obtained with the assumption that the angle between the generatrix of the elastic surface of the membrane and the plane of its fixing in the body is small.

The dependence has the form:

$$A_e = \frac{\pi \cdot D^2}{12} \cdot \left(\beta^2 + \beta + 1 - \frac{x \cdot (1-\beta)\sqrt{4 + 7 \cdot \beta + 4 \cdot \beta^2}}{\sqrt{5 \cdot x_{\max}^2 - 5 \cdot x^2}}\right) \tag{3}$$

where x is the current value of the rigid center displacement from the plane of the diaphragm fixing; $x_{\max}$ - the "maximum" value of the rigid center displacement - the fictitious value at which the diaphragm generator would become rectilinear, and the diaphragm itself would take the form of a truncated cone (Fig. 1).

Dependence (3) indirectly takes into account the structural dimensions, mechanical and elastic properties of the diaphragm material. To determine the effective area from this dependence, it is necessary to determine experimentally the maximum possible displacement of the rigid center each time, which is a disadvantage.

3. Modified dependence of Liktan, proposed in a number of works [2]. The authors use dependence (3) for which $x_{\max}$ is defined as:

$$x_{\max} = x_{st} \cdot \frac{\sqrt{5 \cdot \left(\beta^2 + \beta + 1\right) + (1-\beta) \cdot \left(4 + 7 \cdot \beta + 4 \cdot \beta^2\right)}}{\left(\beta^2 + \beta + 1\right) \cdot \sqrt{5}} \tag{4}$$

where x_{st} is the static characteristic of the diaphragm, which is its free deflection under the action of a given pressure (Fig. 2) [4, 10] and it indirectly takes into account the structural dimensions and mechanical properties of the diaphragm material. The use of this formula also assumes the experimental determination of the diaphragm static characteristic.

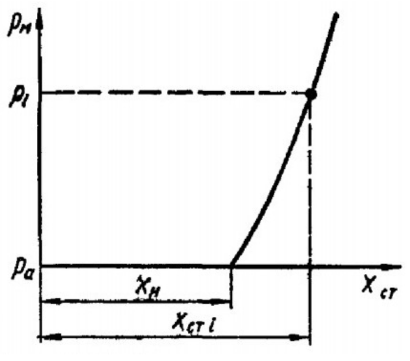

Fig. 2. Typical diaphragm static characteristic

The above dependencies are initially intended for describing thick (several millimeters) diaphragms of powerful actuators. For other applications, for example, sensitive devices, pressure control valves, they give a significant error (more than 40% [5, 6]). Therefore, in order to develop a numerical mathematical model of a pneumatic diaphragm chamber, as a base was chosen a dependence described in a number of works [5, 6], which makes it possible to determine the main characteristics of diaphragm - effective area and stiffness based on comparatively easily and unambiguously measured values - the rigid center displacement, pressures in the chambers, design parameters and physical and mechanical properties of the diaphragm material (membrane diameters and rigid center, the presence and size of the goffers, the thickness of the diaphragm and its material modulus of elasticity etc.). At the same time, depending on the required accuracy, it is possible to take into account or ignore some of these values whose influence is insignificant for the particular application.

Thus, in [5] the following dependence is given for the diaphragm effective area A_e :

$$A_e = \frac{\pi \cdot D^2}{12} \cdot \left(\beta^2 + \beta + 1\right) + \frac{\pi \cdot (D + 2 \cdot d)}{6} \cdot \frac{x}{tg\,\varphi} \tag{5}$$

where l is the length of the goffer; φ - the angle between the chord of the arc AB of the goffer and the tangent to it at the point of intersection with the plane of the diaphragm fixing in the rigid center (Fig. 3).

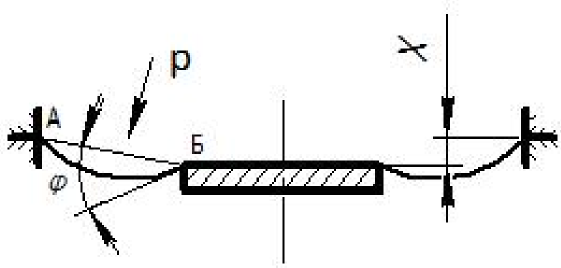

Fig. 3. To the definition of the φ angle

The angle φ is associated with the basic structural dimensions of the diaphragm D, d, l and with the rigid center displacement x by a transcendental dependence [5]:

$$\frac{\sin\varphi}{\varphi} = \frac{\sqrt{\left(\frac{D-d}{2}\right)^2 + x^2}}{l} \tag{6}$$

Since the methods proposed in [5] for solving this equation (graphical one or using predefined tables of angle φ versus $\frac{\sin\varphi}{\varphi}$ dependence) are intended for "manual" calculations, for numerical model adapted to the use with modern CAE systems, the equation (6) has been transformed to the form

$$\varphi = \frac{l \cdot \sin\varphi}{\sqrt{\left(\frac{D-d}{2}\right)^2 + x^2}}\ , \tag{7}$$

and its solution was determined using the Newton-Raphson method

$$\varphi(i+1) = \varphi(i) - f(\varphi(i)) / f^{'}(\varphi(i)) \tag{8}$$

where $f(\varphi(i))$ is the equation (7). This method has good convergence in finding the angle value - in order to find a solution, an average of 3-5 iterations was required.

Based on (5) and (7) a numerical mathematical model was developed and was used to determine the effective area of corrugated diaphragm using CAE system. In Fig. 4 shows the results of calculations in comparison with the experimental data [5, 11]. The comparison shows that the calculated data are in good agreement with the experimental ones, confirming the adequacy of the developed numerical mathematical model.

The fundamental difference between equations (3), (4) and equation (5) is that in the first case the elastic properties of the diaphragm material are taken into account indirectly. Since the diaphragm stiffness is one of the main its features, using equation (5), the "mechanical" part of the diaphragm actuator system of equations needs to be supplemented with a component that takes into account the elasticity of the membrane.

The expression for diaphragm rigidity determining has the form [5]:

$$C = \frac{\pi \cdot p \cdot (D + 2 \cdot d)}{6} \cdot \left[\operatorname{ctg}\varphi + \frac{x^2 \cdot \varphi^3}{l^2 \cdot (\sin\varphi + \varphi \cdot \cos\varphi) \cdot \sin^2\varphi} \right] \tag{9}$$

Graphs of the calculated dependence of the corrugated diaphragm stiffness versus its rigid center displacement for different values of the length of the corrugation are shown in Fig. 5.

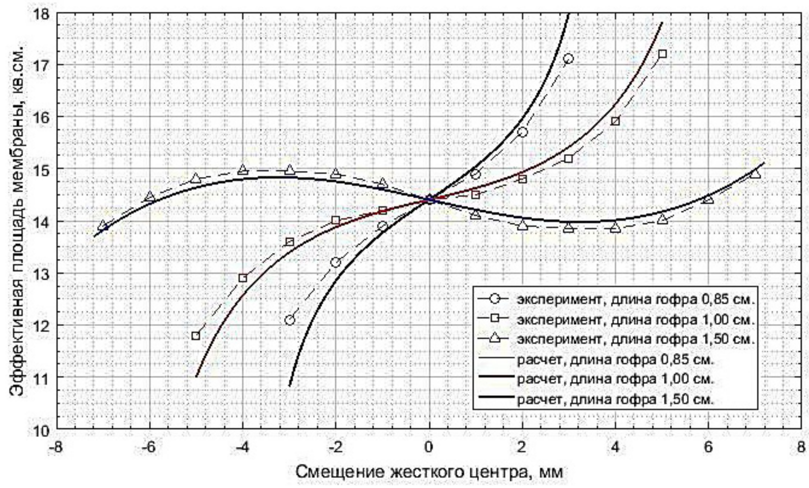

Fig. 4. Dependence of the diaphragm effective area versus rigid center displacement

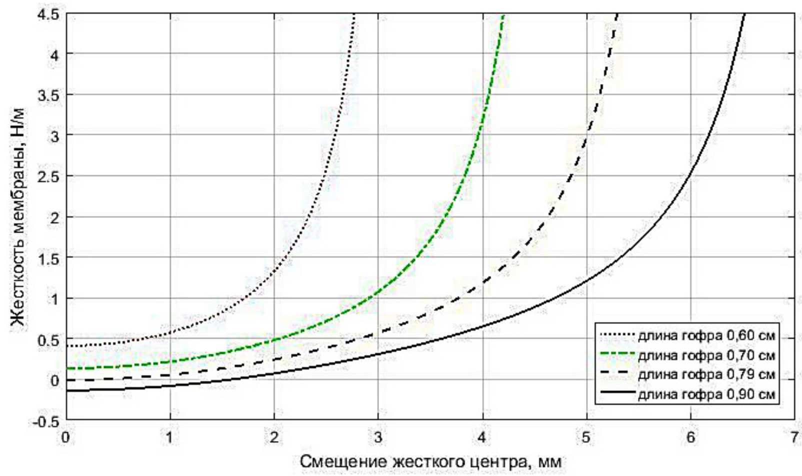

Fig. 5. Dependence of the diaphragm rigidity versus rigid center displacement

The approach described in the paper and based on (5), (7), and (9) to build the model lets take into account and another factors which affect to more or less extent the diaphragm characteristics. For instance, the variation of the effective area due to change of the pressure difference, due to change in configuration of the diaphragm edge radius in the clamping area, or due to applying conical stops which are used to control the value of the effective area, etc. Therefore, the model can easily be adjusted for a particular application by including the most influential factors.

In the case described in the paper, the model was built for the development of a pressure controller for the gas supply system of an internal combustion engine. For this type of application, it was necessary to take into account the variation of the diaphragm effective area due the change in pressure difference.

In [5] the additional deformation of the diaphragm material is suggested to account by correcting the value φ of the angle. The transcendental equation resulted in this approach is shown below

$$\varphi = \frac{l \cdot \sin\varphi}{\sqrt{\left(\frac{D-d}{2}\right)^2 + x^2}} + \frac{p \cdot l}{2 \cdot E \cdot \delta}, \tag{10}$$

where p is the pressure difference on the diaphragm; E is the elasticity modulus of the diaphragm material (Young modulus); δ is the diaphragm thickness. The equation is suggested to be solved with the Newton -Raphson method.

In so doing, the system of equations representing the diaphragm chamber, with the effective area variation due to the rigid center displacement accounted, consists of equations (5) and (10). Fig. 6 shows the results of the effective area computations with these equations together with the experimental measurements [5, 11]. It can be seen that the computation results agree with the experiment.

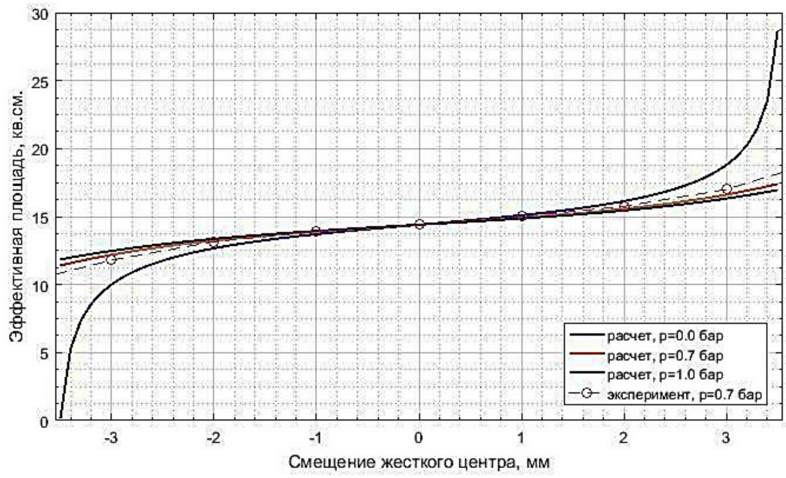

Fig. 6. The variation of the effective area vs. the rigid center displacement and pressure difference

The developed diaphragm chamber model was used in simulation of various pneumatic actuators. Some results of those simulations are presented in the figures below.

Fig. 7 shows the steady-state characteristic of the diaphragm-spring assembly with the following parameters:

Clamping diameter - 6 cm;
Rigid center diameter - 4.8 cm;
Material elasticity modulus - 75e5 N/m^2;
Diaphragm thickness - 0.45 mm;
Spring preload - 995 N;
Spring rate - 150000 N/m

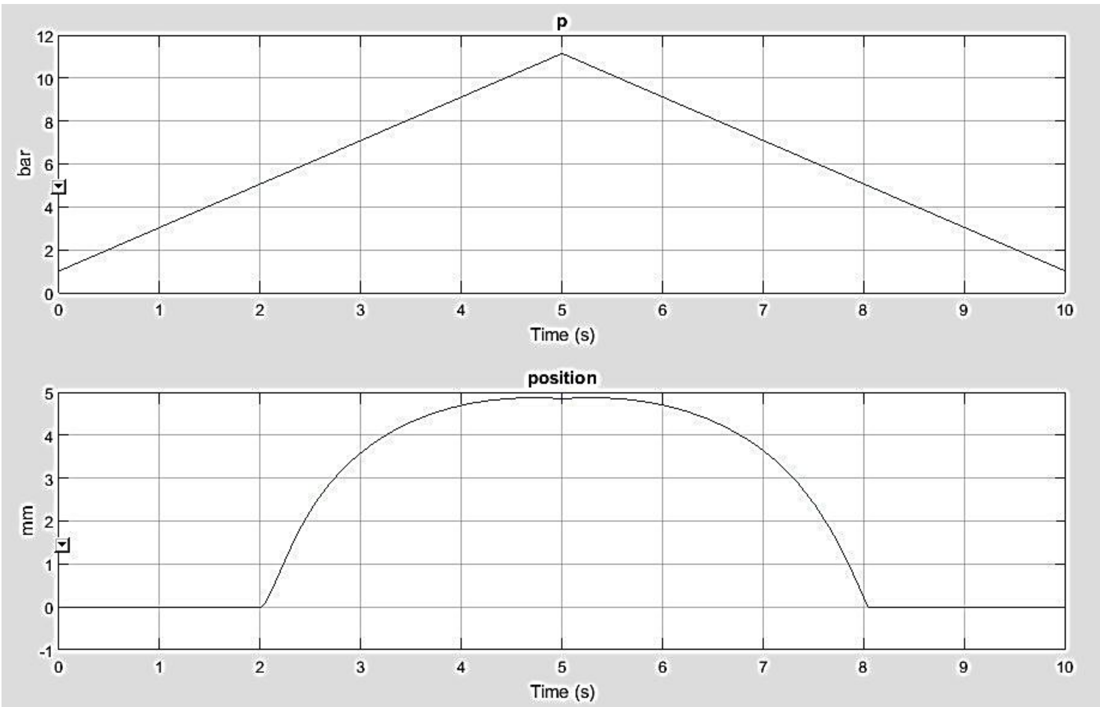

Fig. 7. Steady-state characteristic of the diaphragm actuator

The rigid center remains still up to the moment the pressure difference reaches approximately 5 bar. At this moment, the force due to pressure difference overcomes the spring preload force what cases the center to start moving. The maximum displacement is about 5 mm. The relationship between the displacement and the pressure differential is non-linear due to non-linearity of the diaphragm stiffness.

Fig. 8 shows the results of simulation of a two-staged pressure regulator. The input signal is a pressure step signal of 200 bar applied to the first stage of the regulator. The output line of the regulator is connected to the orifice with

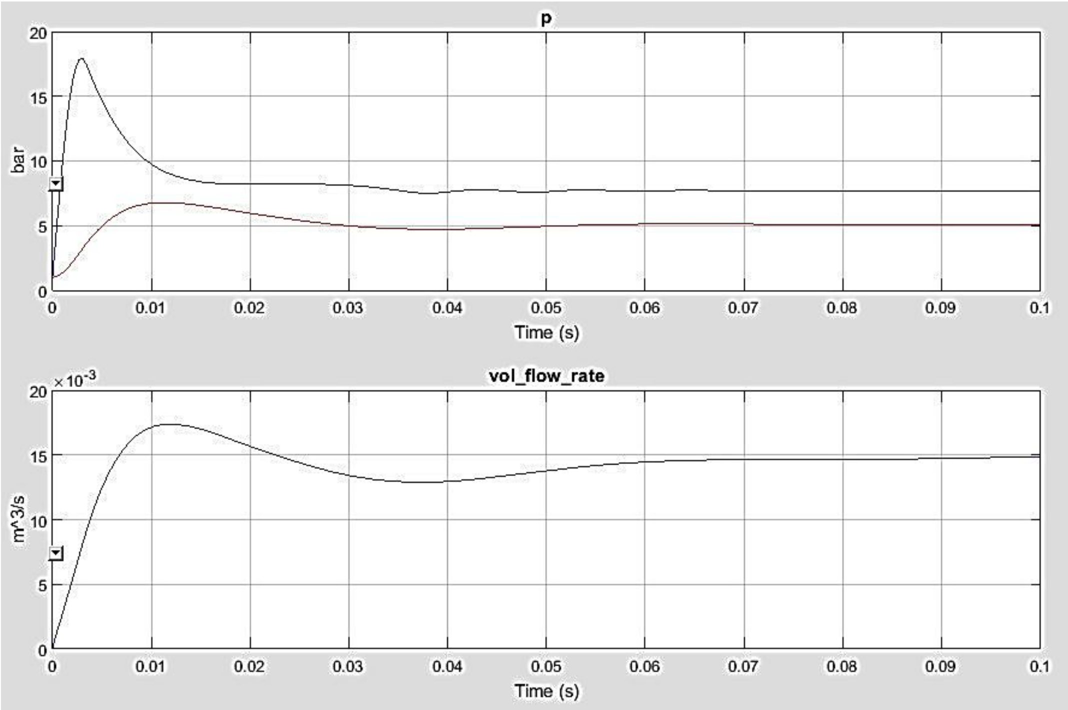

Fig. 8. Two-staged pressure regulator simulation results

0.19 cm^2 effective area. Both diaphragms of the regulator are of the same modulus of elasticity and have the same thickness.

After the step signal has been applied, the first and the second stages of the regulator react in sequence. The transient response is about 0.075 s. Tpe steady state values of the pressures are about 8 bar after the first stage and 5 bar after the second stage. The steady state value of the volumetric flow rate is 15 m^3/s.

The above simulation results show that the developed models can be used for the development and design of pneumatic actuators in various applications.

Conclusions:

1. The numerical model of a pneumatic diaphragm chamber considered as a part of a complex pneumatic actuator was developed. The model can be used in numerical analyses and design of various pneumatic actuation systems such as automobile barking and suspension systems, machine tool devices, pressure regulators, etc.
2. The effective area in the model is determined as the function of easily measurable and normally readily available in catalogs parameters such as the rigid center displacement, pressure differential in the diaphragm chambers, diaphragm thickness, modulus of elasticity, etc.
3. The accuracy of the model was tested by comparing to the experimental results. The model was also tested in simulation of various pneumatic actuators and shown good agreement.

Розробка чисельної моделі пневматичного мембранного приводу

О.О. Коваленко

Аннотація. *Робота присвячена розробці та дослідженню чисельної математичної моделі пневматичної мембранної камери - складової частини моделі пневматичного мембранного приводу, використовуваної в численних моделях більш складних пристроїв (гальмівні системи і підвіски автомобілів, затискні пристрої в оснащенні металорізальних верстатів, регулятори тиску та ін.).*

Для розробленої чисельної моделі ефективна площа мембрани визначається як функція порівняно легко і однозначно вимірюваних фізичних величин - переміщення жорсткого центра, тисків в порожнинах, конструктивних параметрів і фізико-механічних параметрів мембрани (товщина і модуль пружності мембранного полотна).
В роботі перевірена адекватність розробленої моделі, а також наведені результати використання цієї моделі при моделюванні в системах автоматизованого проектування ряду тестових прикладів - мембранних приводів.
Наведені результати моделювання показують, що розроблені моделі адекватно відображають робочі процеси і можуть бути використані для проектування мембранних приводів різного призначення.

Ключові слова: *мембрана; привід; чисельна математична модель; ефективна площа мембрани; нелінійна жорсткість мембрани.*

Разработка численной модели пневматического мембранного привода

А.А. Коваленко

Аннотация. *Настоящая работа посвящена разработке и исследованию численной математической модели пневматической мембранной камеры – составной части модели пневматического мембранного привода, используемой в численных моделях более сложных устройств (тормозные системы и подвески автомобилей, зажимные устройства в оснастке металлорежущих станков, регуляторы давления и др.).*
Для разработанной численной модели эффективная площадь мембраны определяется как функция сравнительно легко и однозначно измеряемых физических величин – перемещения жесткого центра, давлений в полостях, конструктивных параметров и физико-механических параметров мембраны (толщина и модуль упругости мембранного полотна).
В работе проверена адекватность разработанной модели, а также приведены результаты использования этой модели при моделировании в системах автоматизированного проектирования ряда тестовых примеров – мембранных приводов.
Приведенные результаты моделирования показывают, что разработанные модели адекватно отражают рабочие процессы и могут быть использованы для проектирования мембранных приводов различного назначения.

Ключевые слова: *мембрана; привод; численная математическая модель; эффективная площадь мембраны; нелинейная жесткость мембраны.*

References

1. Lichtens, J. (1939), "Calcul de la surface utile des membranes des det deuers", Compt. Rend. Deanc. Acad. Sci, 209, no. 4.
2. Gerts, E.V., Kreinin, G.V. (1960), *Teoria i raschet silovih pnevmaticheskih ustroistv*, izd-vo AN SSSR, Moscow, Russia.
3. Kondrashov, Y.I. (2011), Konstrukcii I proektirovanie agregatov i system, Samar. gos. aerokosm. un-t im, S. P. Koroleva (nac. issled. un-t), Elektron, Tekstovie i graf. dan. (39,8 MB), 1 el. opt. disk (CD-ROM), Samara, Russia.
4. Gerts, E.V., Kreinin, G.V. (1975), *Raschet pnevmoprivodov,* "Mashinostroenie", Moscow, Russian.
5. Vodianik, V.I. (1974), *Elastichnie membrane,* "Mashinostroenie", Moscow, Russian.
6. Bukharin, N.A. (1950), *Tormoznie sistemi avtomobilei,* Mashgiz, Moscow, Russian.
7. Fedorets, V.A., Pedchenko, M.N., Pichko, A.F. Peresad'ko, Y.V. and Lisenko, V.S. (1987), *Gidroprivodi i gidropnevmoavtomatika stankov*, in Fedortsa, V.A.(ed.), Vishcha shk, Kyiv, Ukraina.
8. Fojtášek, K. and Kopáček, J. (2011), "Pneumatic motors with elastic working elements and their mathematica modeling", *Journal of the 21. International conference on hydraulic and pneumatic*, 171.
9. Schomburg, W.K. (2011), "Introduction to Microsistem Design", RWTHedition, *Springer-Verlag Berlin Heidelberg*, DOI 10.1007/978-3-642-19489-4_1.
10. Fojtášek, K., Dvořák, L., Kopáček J. (2012), Měření statických charakteristik pneumatických motorů s elastickými pracovními členy, *Hydraulika a pneumatika*, 1-2, p.9,
11. Fojtášek, K. (2013), Modelování a experimentální ověření chování elastických pneumatických pracovních prvků, Disertační práce, Ostrava.