

# Substitution of equations for evaluation of energy consumption in rotor-stator mixers

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**Abstract.** In the presented publication, an attempt to develop the theory of mixing by considering rotor-stator mixers from the standpoint of mechanical mixing devices was made.

The results of the research on energy consumption are given in the form of analytically substituted expressions for determining power and pressure difference as well as a flow rate in a stage of rotor-stator mixer composed of a pair of the perforated rotor and stator elements with a gap between them, depending on the features of the design, dynamic characteristics of the rotor and flows. The inter-relationships between power, pressure difference, and flow rate in the stages of rotor-stator mixers are established. This makes it possible to define the characteristics of mixers, carry out their calculations and reasonably accept the rational design and processing parameters. The peculiarities of the components in obtained equations are indicated. Partial cases of the equations for power consumption in stages of rotor-stator-mixers operated in pulse and impulse modes are considered. The invariant form of obtained equations would help to ease the scaling of rotor-stator mixers.

**Keywords:** rotor-stator mixer, power, pressure, flow rate.

## Introduction

Rotor-stator mixers (RSM), effective equipment for fluent media processing, are widely used in industry, constantly gaining new implementations in the latest technologies and modernization of existing production facilities. This leads to steady scientific and practical interest in the improvement of the design and development of the theory of these devices, as well as the processes carried out with their use [1–5].

Evaluation of energy consumption is an important part of RSM calculations. One of the most useful solutions for this is to divide the energy supplied by the rotor to the processed media into rotational and pumping components and also an additional component related to the specificity of mixing apparatus design and operation (e.g. energy losses in the end gap between the rotor and the

body, energy losses at the entrance and exit, in bearings, etc.) [1, 2, 5]. This approach has become increasingly widespread and has recently been successfully applied to various designs of rotor-stator devices [6, 7].

A fairly known example of the implementation of the mentioned approach is demonstrated with equation (1) proposed in [8], according to which for the RSM stage, composed of a pair of perforated rotor and stator elements with a gap between them, the power consumption can be found as:

$$N = C \operatorname{Re}^{-\alpha_1} \left( \frac{a_1 z_1 a_2 z_2}{\bar{D}^2} \right)^{\alpha_2} \left( \frac{\delta}{\bar{D}} \right)^{\alpha_3} \left( \frac{s_1 + s_2}{\bar{D}} \right)^{\alpha_4} \times \left( 1 + BQ \frac{a_1 z_1 + a_2 z_2}{a_1 z_1 a_2 z_2 h w} \right) \rho S w^3, \quad (1)$$

where  $C$ ,  $B$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$  are empirical coefficients in equation (1);  $\operatorname{Re}$  is Reynolds number

$$\operatorname{Re} = \frac{\rho n \bar{D}^2}{\mu},$$

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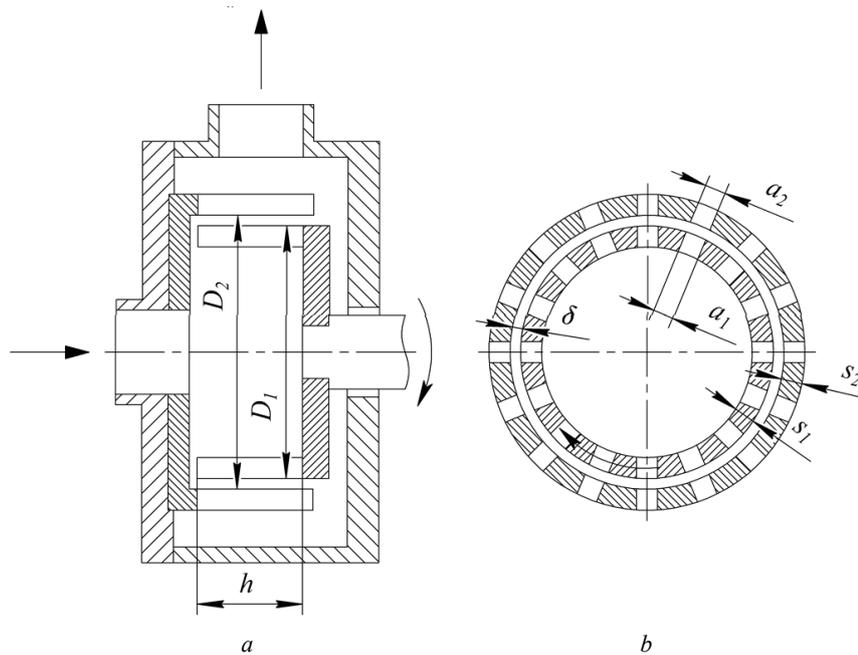


Fig. 1. RSM assembly: a) – longitudinal section; b) – cross-section of the RSM stage

$\rho$  and  $\mu$  are density and dynamic viscosity of processed medium;  $n$  and  $w$  are rotation speed and peripheral velocity of the rotor;  $\bar{D}$  is mean diameter of the gap between the rotor (diameter  $D_1$ ) and stator (diameter  $D_2$ );  $a_1$  and  $a_2$  are width of slots in rotor and stator;  $z_1$  and  $z_2$  are number of slots in rotor and stator;  $\delta$  is width of the gap between rotor and stator;  $s_1$  and  $s_2$  are wall thickness of rotor and stator;  $h$  is effective height of slots;  $S$  is cylindrical surface with a height  $h$ ;  $Q$  is volumetric flow rate. (A schematic representation of the in-line variant of RSM with the designation of sizes used in the description is shown in Fig. 1).

In equation (1) the power consumption on hydrodynamic friction in the end gap between the rotor and the body of the apparatus, as well as consumptions, not directly related to the RSM stage are not taken into account.

### Substitution of equations

Now let us express the solution for equations of fluid motion in functional form according to the theory of mixing in application to mechanical devices [9, 10], using generalized variables:

$$Eu = f(Re, Ho, Fr), \quad (2)$$

where  $Eu$ ,  $Re$ ,  $Ho$ ,  $Fr$  are Euler, Reynolds, homochronism, and Froude numbers.

Energy consumption in the RSM stage is determined by properties of the processed medium, dynamics of the rotor, design features of rotor and stator, and their layout.

Since the rotor is the only moving element in the RSM stage, it is possible to take its velocity as a characteristic parameter that influences the dynamics of fluid motion. Having determined the diameter of rotor ( $D$ ) which faces the gap between rotor and stator as a characteristic size, we can replace this velocity with a proportional value

$$w \sim Dn. \quad (3)$$

Significant rotor velocities, relative smallness of spaces in the RSM stage, and flow redistribution in the stage lead to a considerable decrease in the influence of gravity force on the motion of processed media, which makes it possible to exclude the Froude number from the equation (2).

Let us suppose that in the RSM stage the power transmitted to processed media ( $N$ ) is equal to the sum of the rotational ( $N_\Omega$ ) and the pumping ( $N_Q$ ) components:

$$N = N_\Omega + N_Q. \quad (4)$$

We may say that the rotational component of the transmitted power is:

$$N_\Omega = Fw,$$

where  $F$  is the force used to provide the given rotor velocity.

According to [9], the pressure difference can be replaced by a proportional value:

$$\Delta p_{\Omega} \sim \frac{N_{\Omega}}{wS} \sim \frac{N_{\Omega}}{nD^2h}, \quad (5)$$

where  $S$  is the surface on which the force  $F$  is distributed (for cylindrical surfaces  $S \sim Dh$ ).

Considering (3) and (5), let us express the Euler number as follows:

$$Eu_{\Omega} = \frac{N_{\Omega}}{\rho n^3 D^4 h}, \quad (6)$$

whence

$$N_{\Omega} = Eu_{\Omega} \rho n^3 D^4 h. \quad (7)$$

To determine the pumping component of the transmitted power, according to the theory of centrifugal pumps [11], we will write the pressure difference as follows:

$$\Delta p_Q = \frac{N_Q}{Q}.$$

Thus, as in the case of equation (6), the Euler number can be expressed as

$$Eu_Q = \frac{N_Q}{\rho n^2 D^2 Q}, \quad (8)$$

whence

$$N_Q = Eu_Q \rho n^2 D^2 Q. \quad (9)$$

By substituting (7) and (9) into (4) it can be found that

$$N = Eu_{\Omega} \left( 1 + \frac{Eu_Q}{Eu_{\Omega}} \frac{Q}{nD^2h} \right) \rho n^3 D^4 h. \quad (10)$$

If we denote that  $Eu_{\Omega} \equiv A$  and  $\frac{Eu_Q}{Eu_{\Omega}} \equiv B$ , it is possible to transform (10) to:

$$N = A \left( 1 + \frac{BQ}{nD^2h} \right) \rho n^3 D^4 h. \quad (11)$$

Equation (11) includes a dimensionless complex:

$$A \left( 1 + \frac{BQ}{nD^2h} \right) \equiv K_N, \quad (12)$$

which is equivalent to the power number.

Then from (11), it is possible to write power consumption as

$$N = K_N \rho n^3 D^4 h. \quad (13)$$

The equations (10)–(12) have been obtained by their reduction to  $Eu_{\Omega}$ . Analogous equations can be written by reducing them to  $Eu_Q$ .

Using (3) and taking  $D$  as a characteristic size, it is possible to write the Reynolds number as

$$Re = \frac{\rho n D^2}{\mu}. \quad (14)$$

The peculiarity of using the homochronism number in the proposed solution is the following. Although the RSM stage operates in stationary mode (except for the start and stop periods), the processed media is subjected to systematically changing influences due to alternating passage of solid and flowing parts while the rotor with its rotational speed is passing solid and flowing parts of the stator. Let us take this into account by determining the time scale as

$$t \sim \frac{a_1 z_1 a_2 z_2}{n D^2}. \quad (15)$$

With (15), the homochronism number can be presented as

$$Ho = \frac{a_1 z_1 a_2 z_2}{D^2} \equiv \Gamma_D. \quad (16)$$

Obviously, (16) does not include any of the dynamic parameters. Thus, the homochronism number turns into a complex geometric similarity ( $\Gamma_D$ ), which characterizes the effect of variable flowing of the RSM stage during the steady motion of the rotor on processed media.

When deriving equations (11), (14), and (16) we did not take into account the distance between rotor and stator (gap width  $\delta$ ) and thickness of their walls (generally denoted as  $s$ ). We need to consider these parameters by introducing simplexes of geometric similarity

$$\Gamma_{\delta} = \frac{\delta}{D} \text{ and } \Gamma_s = \frac{s}{D}.$$

Using equations (12), (14), (16) and simplexes  $\Gamma_{\delta}$  and  $\Gamma_s$  the derivation of equation (2) in final form can be written as

$$K_N = C Re^{\alpha_1} \Gamma_D^{\alpha_2} \Gamma_{\delta}^{\alpha_3} \Gamma_s^{\alpha_4}, \quad (17)$$

where  $C$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$  are empirical coefficients in (17).

Obtained equation (11), as well as (13) or (17), makes it possible to determine the power consumed in the RSM stage.

In some cases, it can be useful to determine pressure difference ( $\Delta p$ ) for flow passing through the RSM stage.

For this, taking into account (8),  $K_N$  can be written as

$$K_N = A + \frac{\Delta p Q}{\rho n^3 D^4 h}, \quad (18)$$

whence

$$\Delta p = (K_N - A) \frac{\rho n^3 D^4 h}{Q} = K_{\Delta p} \frac{\rho n^3 D^4 h}{Q}, \quad (19)$$

where  $K_{\Delta p} \equiv (K_N - A)$ .

Equation (19) in an analytical form shows the pressure-flow characteristic of the RSM stage.

If there is a need to estimate the flow rate in the RSM stage, the values necessary for this can be obtained from (18) or from (12).

Certainly, the use of the obtained equations for applied calculations requires the experimental determination of empirical coefficients.

## Results and discussion

### I. Peculiarities of the obtained equations parts.

$K_N$  number. Considering the similarity of operation of RSM to centrifugal pumps, it can be accepted  $Q \sim nD^3$  [11], then from (12)

$$K_N = A \left( 1 + B \frac{D}{h} \right). \quad (20)$$

At the same time, the dynamic parameters of the rotor ( $n$ ) and flow ( $Q$ ) are excluded from the expression  $K_N$ , which makes the equation simpler, but also less informative.

If  $h = D$ , then the equation for  $K_N$  turns to

$$K_N = A \left( 1 + \frac{BQ}{nD^3} \right),$$

from (12), or to

$$K_N = A(1 + B), \quad (21)$$

from (20).

Since the right part of the equation (21) includes only constants, the equation for power consumption could be turned into a form, as it is usually presented for mechanical mixing devices ([9, 10], and others):

$$N = K_N \rho n^3 D^5.$$

Actually, in general, RSM  $h$  differs from  $D$  and exclusion of  $h$  must be taken into account, for example, by entering into the right side of (17) the simplex

$$\Gamma_h = \frac{h}{D}.$$

as an additional power multiplier. However, such an approach complicates the equation and requires more data to obtain empirical coefficients. At the same time, it is worth noting that the specifics of the design and dynamics of flows in RSM allow us to assume a more or less uniform distribution of flows along the effective height within the limits of media motion in local regions of the RSM stage. Thus, it would be preferable to use  $h$  as a part of the equations for  $K_N$ .

$K_{\Delta p}$  number can be found as shown above or may be directly expressed as a function of the same parameters as  $K_N$  number. When  $K_{\Delta p}$  evaluated in both ways its values will coincide with the accuracy of the accepted assumptions and approximations.

Re number retains its expression and essence of modified Reynolds number in the theory of mixing in application to mechanical mixing devices [9, 10].

Complex  $\Gamma_D$ . For the case of rotor and stator perforation with slots (as in Fig. 1), it is possible to write

$$\Gamma_D = k_{1l} k_{2l},$$

where  $k_{1l}$  and  $k_{2l}$  are linear perforation coefficients of rotor and stator:

$$k_{1l} = \frac{a_1 z_1}{l_1} \quad \text{and} \quad k_{2l} = \frac{a_2 z_2}{l_2},$$

where  $l_1 = \pi D_1$  and  $l_2 = \pi D_2$  are rotor and stator circumferences.

$\Gamma_D$  generalized for different types of perforation of rotor and stator can be presented as follows

$$\Gamma_D = k_{1s} k_{2s},$$

where  $k_{1s}$  and  $k_{2s}$  are surface perforation coefficients of rotor and stator.

Simplex  $\Gamma_s$  takes into account the impact of the gap on the hydrodynamics of the RSM stage, which is appeared as a redistribution of flows, changes in their velocities and pressures, and energy dissipation in processed media.

Simplex  $\Gamma_s$  is introduced in a general form: it can be written separately for rotor and stator, or only for the rotor.

From (1),  $\Gamma_s$  could be presented as

$$\Gamma_s = \frac{s_1 + s_2}{D},$$

which can be especially useful for describing RSM with only one stage.

In some cases  $\Gamma_s$  can be excluded from the equation. Thus, for the RSM which operates in pulse mode, it is recommended to enter this simplex [1], while for the RSM which operates in impulse mode the influence of  $\Gamma_s$  is not considered significant [2]. In general, it can not, probably, be taken into account in the case of small thicknesses of the rotor and stator walls, significant pumping of liquid through the RSM stage, and in individual cases of perforation distribution also as other specifics of the rotor and stator design. However, this issue requires further research.

### II. Some partial cases of the resulting equations for power consumption.

From (11)

$$A = \frac{N}{\left(1 + \frac{BQ}{nD^2h}\right) \rho n^3 D^4 h}. \quad (22)$$

It is possible to present  $A$  as a function of dimensionless parameters, similarly to (17), as follows

$$A = C \operatorname{Re}^{\alpha_1} \Gamma_D^{\alpha_2} \Gamma_\delta^{\alpha_3} \Gamma_s^{\alpha_4}, \quad (23)$$

where  $C$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ ,  $\alpha_4$  are empirical coefficients

for equation (23).

Then from (22) and (23)

$$N = C \operatorname{Re}^{\alpha_1} \Gamma_D^{\alpha_2} \Gamma_\delta^{\alpha_3} \Gamma_s^{\alpha_4} \left(1 + \frac{BQ}{nD^2h}\right) \rho n^3 D^4 h. \quad (24)$$

The resulting equation (24) is similar to (1).

Use of  $D$  or  $\bar{D}$  as characteristic sizes in equations (1) or (24) does not change its expressions in essence, so any of them can be taken as the appropriate justification.

For RSM operating in impulse mode [2], if we neglect the power component  $N_Q$  and the influence of the wall thicknesses of the rotor and stator, from (24) it is possible to obtain

$$N = C \operatorname{Re}^{\alpha_1} \Gamma_D^{\alpha_2} \Gamma_\delta^{\alpha_3} \rho n^3 D^4 h, \quad (25)$$

where  $C$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  are empirical coefficients for equation (25).

Equation (25) is similar to that proposed in [12], which was obtained for the case of the same number of identical slots in the rotor and stator. In this case  $\Gamma_D$ , will be

$$\Gamma_D = \frac{az}{D},$$

where  $a$  and  $z$  are numbers of slots in the rotor or stator.

## Conclusions

The resulting dependencies complement the theory of mixing, making it possible to consider RSM from the standpoint of mechanical mixing devices. Their use allows you to determine the main hydrodynamic parameters, compare characteristics and carry out scaling of devices with an arbitrary number, shape, and location of perforation in rotor and stator. This opens up additional opportunities for theoretical and experimental research aimed at improving the design and increasing the efficiency of the RSM.

## References

- [1] M.A. Balabudkin, *Rotorno-pul'sacionnye apparaty v himiko-farmaceuticheskoy promyshlennosti*, Moscow: Medicina, 1983.
- [2] M.A. Promtov, A.Yu. Stepanov and A.V. Aleshin, *Metody rascheta harakteristik rotornogo impul'snogo apparata*, Tambov: FGBOU VPO "TGTU", 2015.
- [3] V.A. Atiemo-Obeng and R. Calabrese, "Rotor-stator mixing devices," in *Handbook of Industrial Mixing*, E.L. Paul, V.A. Atiemo-Obeng and S.M. Kresta, Eds., Hoboken, NJ, USA: Wiley, 2004. ch. 8, pp. 479–505.
- [4] V. Vashisth, K.D.P. Nigam and V. Kumar "Design and development of high shear mixers: Fundamentals, applications and recent progress," *Chem. Eng. Sci.*, vol. 232, article 116296, Mar. 2021, doi: 10.1016/j.ces.2020.116296
- [5] J. Zhang, S. Xu and W. Li "High shear mixers: A review of typical applications and studies on power draw, slot pattern, energy dissipation and transfer properties," *Chem. Eng. and Process.: Process Intensification*, vol. 57–58, pp. 25–41, Jul.-Aug. 2012, doi: 10.1016/j.cep.2012.04.004.
- [6] T.P. John, J.S. Panesar, A. Kowalski, T.L. Rodgers and C.P. Fonte "Linking power and flow in rotor-stator mixers", *Chem. Eng. Sci.*, Vol. 207, pp. 504–515, Nov. 2019, doi: 10.1016/j.ces.2019.06.039.
- [7] B.A. Minnick, J.W. Kim, D.I. Ko, R.V. Calabrese "Flow and power characteristics of an axial discharge rotor-stator mixer", *Chem. Eng. Res. and Design*, Vol. 136, pp. 477–490, Aug. 2018, doi: 10.1016/j.cherd.2018.05.015.
- [8] M.A. Balabudkin, "Calculation of power consumption in rotary-pulsation apparatuses," *Khimiko-Farmatsevticheskii Zhurnal*, No. 3, pp. 124–128, 1977.
- [9] A.G. Kasatkin, *Osnovnye processy i apparaty himicheskoy tekhnologii*, Moscow: Himiya, 1973.
- [10] Z. Shterbachek and P. Tausk, *Peremeshivanie v himicheskoy promyshlennosti*, Leningrad: Gosudarstvennoe nauchno-tekhnicheskoe izdatel'stvo himicheskoy literatury (In Russian), 1963.
- [11] V.A. Zimmickij, A.V. Kaplun, A.N. Papir and V.A. Umov Lopastnye nasosy: spravochnik, Leningrad: Mashinostroenie, 1986.
- [12] M.A. Promtov "Analysis of basic parameters of radial types of pulsed rotary equipment," *Chem. and Petroleum Eng.*, No. 9, pp. 13–15, 2009.

## Обґрунтування рівнянь для оцінки витрат енергії у роторно-пульсаційних апаратах

О.О. Семінський

*Анотація.* У представленій публікації зроблено спробу розвинути теорію перемішування розглянувши роторно-пульсаційні апарати з позицій апаратів з механічними перемішувальними пристроями.

Наведені результати дослідження витрат енергії, виражені у аналітично обґрунтованих виразах для визначення потужності і зміни тиску, а також об'ємної витрати у ступені апарата (під якою розуміється пара з перфорованих елементів ротора і статора, що утворюють між собою радіальний зазор), в залежності від особливостей конструювання конструкції, динамічних характеристик ротора і потоків в апараті. Встановлено взаємозв'язки між потужністю, зміною тиску у ступені і витратою текучого середовища, що дозволяє будувати характеристики ступенів, проводити їх розрахунки, обґрунтовано приймати раціональні параметри конструкцій і режимні параметри їх роботи. Зазначено особливості складових у виразах. Розглянуто частинні випадки одержаних виразів для визначення потужності для апаратів, які працюють в пульсаційному та імпульсному режимах. Інваріантна форма одержаних виразів полегшує масштабування роторно-пульсаційних апаратів.

**Ключові слова:** роторно-пульсаційний апарат, потужність, тиск, витрата.