Mech. Adv. Technol., Vol. 8, No. 1, 2024, pp. 80–86

DOI: 10.20535/2521-1943.2024.8.1(100).297514

UDK 519.6

Study of motion stability of a viscoelastic rod

I. Kostyushko[1] • H. Shapovalov[1]

Received: 29 January 2024 / Revised: 19 February 2024 / Accepted: 2 March 2024

Abstract. *Stability of non-conservatively loaded elastic and inelastic bodies - a classic section of deformable solid mechanics that has been of interest for many years. In this paper, we study the motion stability of a free rod subjected to a constant tracking force on one of its ends. The problem is interesting in practical application, as it can be viewed as a simplified model of a rocket moving under the action of a jet force. The defining ratio of the rod material is the Kelvin-Voigt model. The solution to the problem is presented as a decomposition of the beam function. The number of terms of this expansion is substantiated. The critical load values in the presence and absence of viscosity are determined. It is established that the existence of a non-zero value of the internal viscosity coefficient in the Kelvin-Voigt model leads to a significant reduction in the critical load value compared to the elastic rod model. The given analytical results are confirmed by numerical calculations.*

Keywords: *Kelvin-Voigt model, critical force, viscosity coefficient, stability, beam functions.*

Statement of the problem. Classical results

The paper deals with the problem of determining the critical load of a homogeneous free rod of length l, which is in equally accelerated motion under the action of a tracking force *P*. The material of the rod corresponds to the Kelvin-Voigt model:

$$\sigma = E\left(\varepsilon + \nu\dot{\varepsilon}\right),$$

where σ – stress, ε – strain, ν – relaxation time, and E – elastic modulus.

The equation of small vibrations of a rod is of the form [1]:

$$EI\frac{\partial^4 V}{\partial x^4} + EI\nu\frac{\partial^5 V}{\partial x^4\partial t} + \frac{\partial}{\partial x}\left[P\left(1-\frac{x}{l}\right)\frac{\partial V}{\partial x}\right] + \rho S\frac{\partial^2 V}{\partial t^2} = 0,$$

where $V(x,t)$ is the deviation of the rod point with the coordinate x at the moment of time *t*; *EI* is the stiffness of the section in bending; ρ is the density of the material; S is the cross-sectional area. In the above equation, we make the transition to dimensionless variables (W,ξ,τ) by the formulas:

$$W = \frac{V}{l};\xi = \frac{x}{l};\ \tau = t\frac{1}{l^2}\sqrt{\frac{EI}{\rho S}}\ .$$

We obtain the differential equation of motion of the rod in its final form:

$$\frac{\partial^4 W}{\partial\xi^4} + k\frac{\partial^5 W}{\partial\xi^4\partial\tau} - p\frac{\partial W}{\partial\xi} + p(1-\xi)\frac{\partial^2 W}{\partial\xi^2} + \frac{\partial^2 W}{\partial\tau^2} = 0, \quad (1)$$

where $p = \frac{Pl^2}{EI}$; $k = \nu\sqrt{\frac{EI}{\rho S}}\frac{1}{l^2}$; k and p are the parameters of the problem, respectively, the internal viscosity coefficient and the dimensionless force. The solution of equation (1) must satisfy the boundary conditions:

$$\left.\frac{\partial^2 W}{\partial\xi^2}\right|_{\xi=0} = \left.\frac{\partial^3 W}{\partial\xi^3}\right|_{\xi=0} = 0;\ \left.\frac{\partial^2 W}{\partial\xi^2}\right|_{\xi=1} = \left.\frac{\partial^3 W}{\partial\xi^3}\right|_{\xi=1} = 0. \quad (2)$$

The value $k = 0$ corresponds to the classical case of an elastic rod. When the internal viscosity $k > 0$ is taken into account, the dependence of the critical load p^* in the

✉ *I. Kostyushko*
kostiushkoia5@gmail.com

[1] *Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine*

vicinity of the point $k = 0$ is described by a quadratic function [2], [6], with $\lim_{k \to *0} p^*(k) \neq p^*(0)$. The described phenomenon is known in the literature as the "Ziegler paradox". An explanation of this "paradox" is given in [3]–[5], [7]–[10].

In this paper, we are interested in obtaining the dependence of the critical load p^* on the internal viscosity coefficient *k*. In particular, the existence of a "jump" at a vanishingly small value of *k*. We are also interested in the behavior of p^* as $k \to \infty$.

Problem analysis, calculation formulas

The solution to problems (1), (2) is represented as a series:

$$W(\xi,\tau) = \sum_{i=1}^{N} T_i(\tau) \cdot X_i(\xi), \tag{3}$$

where $X_i(\xi)$ are linear combinations of beam functions:

$$X_i(\xi) = X_i(0) \cdot S_1(\delta_i \cdot \xi) + X_i'(0) \cdot S_2(\delta_i \cdot \xi) + \\ + X_i''(0) \cdot S_3(\delta_i \cdot \xi) + X_i'''(0) \cdot S_4(\delta_i \cdot \xi);$$

$$S_1(\delta_i \cdot \xi) = \frac{1}{2}\left(ch(\delta_i \cdot \xi) + \cos(\delta_i \cdot \xi)\right);$$

$$S_2(\delta_i \cdot \xi) = \frac{1}{2}\left(sh(\delta_i \cdot \xi) + \sin(\delta_i \cdot \xi)\right);$$

$$S_3(\delta_i \cdot \xi) = \frac{1}{2}\left(ch(\delta_i \cdot \xi) - \cos(\delta_i \cdot \xi)\right);$$

$$S_4(\delta_i \cdot \xi) = \frac{1}{2}\left(sh(\delta_i \cdot \xi) - \sin(\delta_i \cdot \xi)\right).$$

Substituting (3) into (2) leads to the following form of the functions $X_i(\xi)$:

$$X_i(\xi) = -\frac{S_3(\delta_i)}{S_2(\delta_i)} \cdot S_1(\delta_i \cdot \xi) + S_2(\delta_i \cdot \xi), \tag{4}$$

where δ_i are solutions of the transcendental equation

$$ch(\delta_i \cdot \xi) \cdot \cos(\delta_i \cdot \xi) = 1, \quad \left(i = \overline{1,N}\right) \tag{5}$$

the roots of which are determined by numerical methods. According to (3), (1), the functions $T_j(\tau)$ $\left(j = \overline{1,N}\right)$ satisfy the differential equation:

$$\sum_{j=1}^{N} \frac{d^2 T_j(\tau)}{d\tau^2} \cdot X_j(\xi) + k \sum_{j=1}^{N} \frac{dT_j(\tau)}{d\tau} \cdot X_j^{(IV)}(\xi) + \\ + \sum_{j=1}^{N} T_j(\tau) \left[X_j^{(IV)}(\xi) - p\frac{dX_j(\xi)}{d\xi} + p(1-\xi)\frac{d^2 X_j(\xi)}{d\xi^2} \right] = 0. \tag{6}$$

Equation (6) is multiplied by the functions $X_i(\xi)$ $\left(i = \overline{1,N}\right)$ and the resulting relations are integrated over the variable ξ along the interval from 0 to 1.

Considering the property of beam functions:

$$\frac{d^4 X_i(\xi)}{d\xi^4} = \delta_i^4 \cdot X_i(\xi);$$

$$\int_0^1 X_i(\xi) \cdot X_j(\xi) d\xi = 0 \quad \text{if } i \neq j,$$

we arrive at a closed system of *N* linear homogeneous differential equations with respect to the unknown functions $T_i(\tau)\left(i = \overline{1,N}\right)$:

$$\frac{d^2 T_i(\tau)}{d\tau^2} + k \cdot \delta_i^4 \cdot \frac{dT_i(\tau)}{d\tau} + \delta_i^4 \cdot T_i(\tau) + p \cdot \sum_{j=1}^{N} d_{i,j} T_j(\tau) = 0, \tag{7}$$

where $d_{i,j}$ are the designated numbers:

$$d_{i,j} = \frac{\int_0^1 (1-\xi) \cdot X_i(\xi) \cdot \frac{d^2 X_j(\xi)}{d\xi^2} d\xi - \int_0^1 X_i(\xi) \cdot \frac{dX_j(\xi)}{d\xi} d\xi}{\int_0^1 X_i^2(\xi) d\xi}.$$

For the system (7), the corresponding characteristic equation for an arbitrary value of *N* is of the form:

$$\begin{vmatrix} \Omega_1 & pd_{1,2} & pd_{1,3} & pd_{1,4} & \cdots & pd_{1,N} \\ pd_{2,1} & \Omega_2 & pd_{2,3} & pd_{2,4} & \cdots & pd_{2,N} \\ pd_{3,1} & pd_{3,2} & \Omega_3 & pd_{3,4} & \cdots & pd_{3,N} \\ pd_{4,1} & pd_{4,2} & pd_{4,3} & \Omega_4 & \cdots & pd_{4,N} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ pd_{5,1} & pd_{5,2} & pd_{5,3} & pd_{5,4} & \cdots & \Omega_N \end{vmatrix} = 0, \tag{8}$$

where

$$\Omega_i = \omega^2 + k \cdot \delta_i^4 \cdot \omega + \delta_i^4 + pd_{i,i}, \quad \left(i = \overline{1,4}\right).$$

The case of an elastic rod

Let us consider the question of the appropriate number of terms *N* in the expansion (3).

Let us set the internal viscosity coefficient $k = 0$. Determine the value of the critical load p^*, at which the solution of system (7) will be stable.

For $N = 2$, the characteristic equation (8) can be represented as follows:

$$\omega^4 + b_1(p) \cdot \omega^2 + b_2(p) = 0, \tag{9}$$

where $b_i(p)$ $(i = 1,2)$ are known polynomials with respect to the parameter p of the 1st and 2nd degree, respectively.

According to the Rausch-Hurwitz criterion, the solution of system (7) will be stable if the roots of the biquadratic equation (9) contain only negative real parts. This is possible if the equation

$$y^2 + b_1(p)\cdot y + b_2(p) = 0$$

has two negative real roots:

$$\begin{cases} (b_1(p))^2 - 4b_2(p) > 0; \\ b_1(p) > 0; \\ b_2(p) > 0. \end{cases} \quad (10)$$

The solution of (10) is the set $0 < p < 116.0969$. So, $p^*\big|_{N=2} \approx 116.10$.

When $N = 3$, equation (8) takes the form:

$$\omega^6 + \bar{b}_1(p)\cdot\omega^4 + \bar{b}_2(p)\cdot\omega^2 + \bar{b}_3(p) = 0, \quad (11)$$

$\bar{b}_i(p)$ $\left(i = \overline{1,3}\right)$ – the defined polynomials of the i-th degree, respectively. Substitution of $\omega^2 = y$ in (11) leads to a cubic equation:

$$y^3 + \bar{b}_1(p)\cdot y^2 + \bar{b}_2(p)\cdot y + \bar{b}_3(p) = 0. \quad (12)$$

The condition for the existence of real roots of equation (12) is reduced to solving the inequality:

$$\frac{q^2(p)}{4} + \frac{r^3(p)}{27} \le 0, \quad (13)$$

where

$$q(p) = \frac{2}{27}\bar{b}_1^3(p) - \frac{1}{3}\bar{b}_1(p)\bar{b}_2(p) + \bar{b}_3(p);$$

$$r(p) = \bar{b}_2(p) - \frac{1}{3}\bar{b}_1^2(p).$$

To fulfil the condition of negative real parts of the roots of equation (12), we use the well-known Rauss-Hurwitz criterion: the main diagonal determinants of the Hurwitz matrix:

$$\begin{pmatrix} \bar{b}_1(p) & 1 & 0 \\ b_3(p) & \bar{b}_2(p) & \bar{b}_1(p) \\ 0 & 0 & \bar{b}_3(p) \end{pmatrix}$$

must be positive:

$$\begin{cases} \bar{b}_1(p) > 0; \\ \bar{b}_1(p)\bar{b}_2(p) - \bar{b}_3(p) > 0; \\ \bar{b}_3(p) > 0. \end{cases} \quad (14)$$

The solution of the system of inequalities (13), (14) is the set $0 < p < 109{,}9842$. So, $p^*\big|_{N=3} \approx 109.98$. Thuse $p^*\big|_{N=3} \ne p^*\big|_{N=2}$.

Let us consider the case of $N = 4$. The characteristic equation (7) is reduced to the form

$$\omega^8 + \tilde{b}_1(p)\cdot\omega^6 + \tilde{b}_2(p)\cdot\omega^4 + \tilde{b}_3(p)\cdot\omega^2 + \tilde{b}_4(p) = 0, \quad (15)$$

where $\tilde{b}_i(p)\left(i = \overline{1,3}\right)$ are the designated polynomials of the i-th degree, respectively.

Equation (15) has imaginary roots if equation

$$y^4 + \tilde{b}_1(p)\cdot y^3 + \tilde{b}_2(p)\cdot y^2 + \tilde{b}_3(p)\cdot y + \tilde{b}_4(p) = 0 \quad (16)$$

has real negative roots. The substitution $y = z - \frac{\tilde{b}_1(p)}{4}$ leads to an incomplete equation (16):

$$z^4 + \tilde{p}(p)\cdot z^2 + q(p)\cdot y + r(p) = 0, \quad (17)$$

where

$$\tilde{p}(p) = \frac{1}{8}\left[8\cdot\tilde{b}_2(p) - 3\cdot\tilde{b}_1^2(p)\right];$$

$$q(p) = \frac{1}{8}\left[8\cdot\tilde{b}_3(p) - 4\cdot\tilde{b}_1(p)\tilde{b}_2(p) + \tilde{b}_1^3(p)\right];$$

$$r(p) = \frac{256\cdot\tilde{b}_4(p) - 64\cdot\tilde{b}(p)\tilde{b}_3(p) + 16\cdot\tilde{b}_1^2(p)\tilde{b}_2(p) - 3\cdot\tilde{b}_1^4(p)}{256}.$$

The solution of the incomplete equation (17) can be determined by solving the cubic residual:

$$t^3 + B_1(p)\cdot t^2 + B_2(p)\cdot t + B_3(p) = 0, \quad (18)$$

where

$$B_1(p) = -2\cdot\tilde{p}(p);$$

$$B_2(p) = \tilde{p}^2(p) - 4\cdot r(p);$$

$$B_3(p) = q^2(p).$$

The roots $z_i\left(i = \overline{1,3}\right)$ of equation (17) are determined through the roots of the resolvent (18) t_i by the following relations:

$$t_1 = (z_1 + z_2)(z_3 + z_4);$$

$$t_2 = (z_1 + z_3)(z_2 + z_4);$$

$$t_3 = (z_1 + z_4)(z_2 + z_3).$$

If the roots of z_i are real numbers, then the roots of t_i will also be real. The condition for the existence of real roots of equation (18):

$$\frac{Q^2(p)}{4} + \frac{R^3(p)}{27} \le 0, \quad (19)$$

where

$$Q(p) = \frac{2}{27}B_1^3(p) - \frac{1}{3}B_1(p)B_2(p) + B_3(p);$$

$$R(p) = B_2(p) - \frac{1}{3}B_1^2(p).$$

The condition for the existence of negative real parts of the roots of equation (16) is the fulfillment of the Hurwitz criterion:

$$\begin{cases} \tilde{b}_1(p) > 0; \\ \begin{vmatrix} \tilde{b}_1(p) & 1 \\ \tilde{b}_3(p) & \tilde{b}_2(p) \end{vmatrix} > 0; \\ \begin{vmatrix} \tilde{b}_1(p) & 1 & 0 \\ \tilde{b}_3(p) & \tilde{b}_2(p) & \tilde{b}_1(p) \\ 0 & \tilde{b}_4(p) & \tilde{b}_3(p) \end{vmatrix} > 0; \\ \tilde{b}_4(p) > 0. \end{cases} \quad (20)$$

The solution to the system of inequalities (19), (20) is the set: $0 < p < 109,978$. Consequently, the value of the critical load $p^*\big|_{N=4} \approx p^*\big|_{N=3} \approx 109,98$.

In the following, we will limit ourselves to three terms of $N = 3$ in the expansion (3).

The case of a viscoelastic rod

If $k > 0$, the characteristic equation (8) of system (7) is reduced to the form

$$\omega^6 + b_1(k,p)\omega^5 + b_2(k,p)\omega^4 + b_3(k,p)\omega^3 + \\ + b_4(k,p)\omega^2 + b_5(k,p)\omega + b_6(k,p) = 0, \quad (21)$$

where

$$\begin{cases} b_1(k,p) \approx 18921 \cdot k; \\ b_2(k,p) \approx 6,4820 \cdot 10^7 \cdot k^2 - 78,629 \cdot p + 18922; \\ b_3(k,p) \approx 1,2964 \cdot 10^8 \cdot k + 2,7831 \cdot 10^{10} \cdot k^3 - \\ \quad - 6,7426 \cdot 10^5 \cdot k \cdot p; \\ b_4(k,p) \approx 8,3492 \cdot 10^{10} k^2 - 6,7426 \cdot 10^5 p + \\ \quad + 1613,1 p^2 + 6,4820 \cdot 10^7 - 6,0463 \cdot 10^8 k^2 p; \\ b_5(k,p) \approx 8,3492 \cdot 10^{10} k + 4,2064 \cdot 10^6 \cdot k \cdot p^2 - \\ \quad - 1,2093 \cdot 10^9 \cdot k \cdot p; \\ b_6(k,p) \approx -6,0463 \cdot 10^8 \cdot p + 2,7831 \cdot 10^{10} - \\ \quad - 7184,2 \cdot p^3 + 4,2065 \cdot 10^6 \cdot p^2. \end{cases} \quad (22)$$

For equation (21), the Hurwitz matrix has the following form:

$$\begin{pmatrix} b_1 & 1 & 0 & 0 & 0 & 0 \\ b_3 & b_2 & b_1 & 1 & 0 & 0 \\ b_5 & b_4 & b_3 & b_2 & b_1 & 1 \\ 0 & b_6 & b_5 & b_4 & b_3 & b_2 \\ 0 & 0 & 0 & b_6 & b_5 & b_4 \\ 0 & 0 & 0 & 0 & 0 & b_6 \end{pmatrix}.$$

For a fixed value of k , the Rausch-Gurwitz criterion can be used to determine the value of the critical load p^* according to the solution of the system of inequalities: $\Delta_i > 0 \left(i = \overline{2,6}\right)$, Δ_i – are the main determinants of the given matrix $\left(\Delta_1 = b_1(k,p) > 0\right)$. For example, when $k = \{0,05;\ 0,15;\ 0,25\}$, the corresponding values of the critical load $p^* \approx \{100,2;\ 103,5;\ 103,86\}$.

Fig. 1 shows the dependence $p^* = p^*(k)$. The behavior of the function at $k \to 0$ and at $k \to \infty$ is interesting. The expected behavior of this function: $\lim_{k\to 0} p^*(k) = p^*\big|_{k=0}$. But, as the calculations show, even with an extremely small value of the viscosity coefficient $k = 10^{-10}$, the value of the critical load $p^*\big|_{k=10^{-10}} \approx 88,213$ is significantly less than the value $p^*\big|_{k=0} \approx 109,98$. This phenomenon is called the destabilization paradox [4]. The other limiting value of the critical load $\lim_{k\to\infty} p^*(k) \approx 104$ is also less than the determined value $p^*\big|_{k=0} = 109,98$. That is the existence of any viscosity, even however small, leads to a decrease in the stability limit by a finite amount.

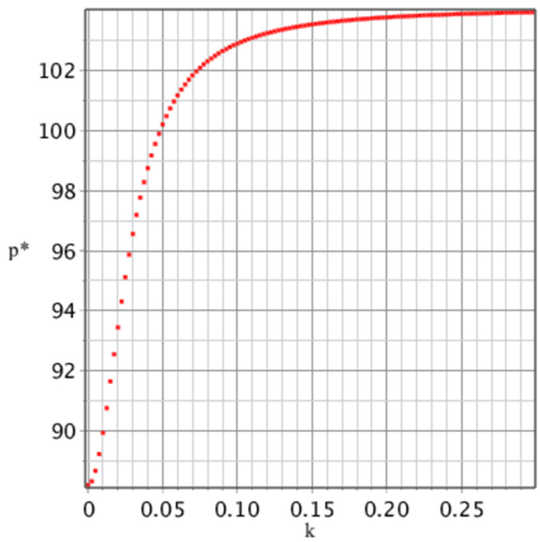

Fig. 1. Dependence of the critical force on the viscosity coefficient

Numerical studies

To illustrate the results obtained, we present a numerical solution of the system of differential equations (6) by the Runge-Kutta method of order 7–8.

Figs. 2–4 show the graphs of the functions $T_i(\tau)\left(i=\overline{1,3}\right)$ at $k=0;\ p=20<p^*\big|_{k=0}\approx 109{,}98$ and arbitrary initial conditions: $T_1(0)=T_2(0)=T_3(0)=T_1'(0)=$ $=T_2'(0)=0{,}01;\ T_3'(0)=-0{,}01$.

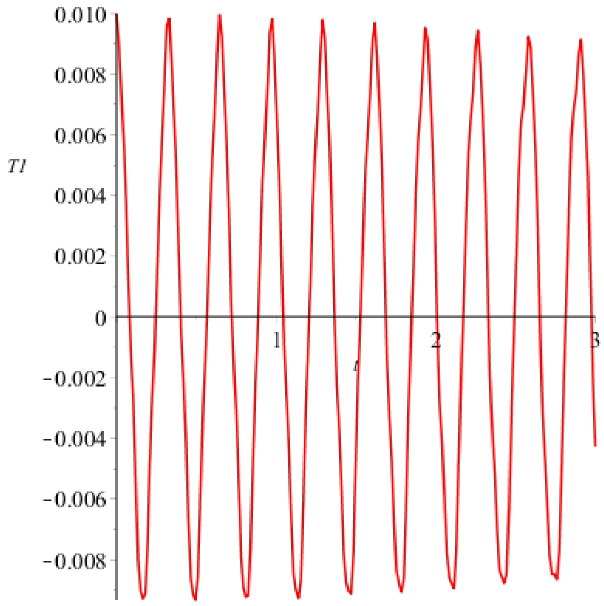

Fig. 2. Graph of the function $T_1(\tau)$ at p = 20

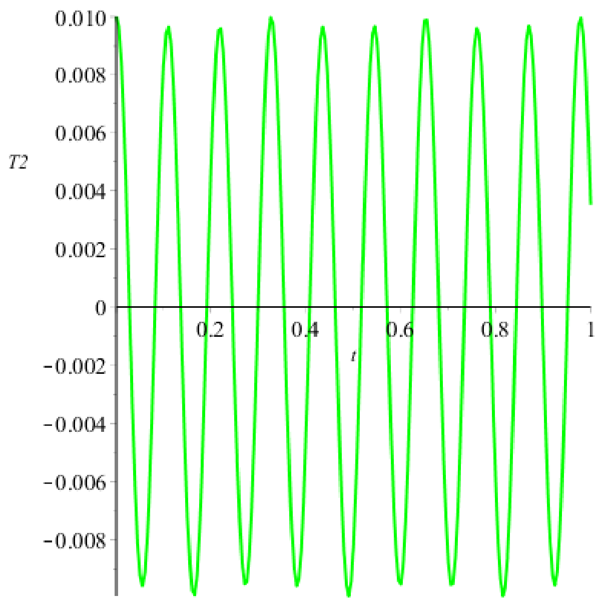

Fig. 3. Graph of the function $T_2(\tau)$ at p = 20

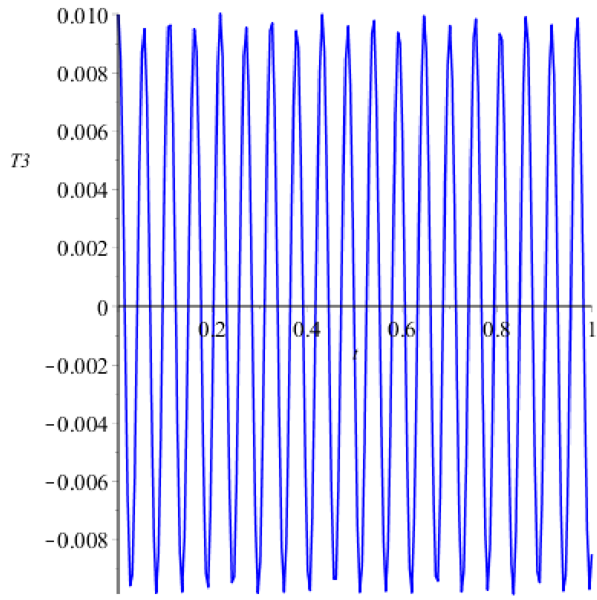

Fig. 4. Graph of the function $T_3(\tau)$ at p = 20

In Figs. 5–7 show the graphs of the functions $T_i(\tau)\left(i=\overline{1,3}\right)$ at $k=0;\ p=150>p^*\big|_{k=0}\approx 109{,}98$.

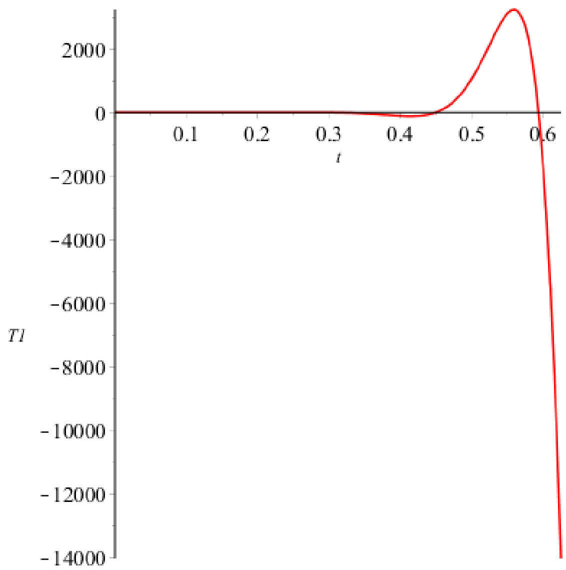

Fig. 5. Graph of the function $T_1(\tau)$ at p = 150

At a relatively small value of $\tau\approx 0{,}63$ for $p>p^*\approx 109{,}98$, the functions $T_i(\tau)$ $\left(i=\overline{1,3}\right)$ tend to infinity, indicating the solution's instability. It is shown that changing the initial conditions does not affect the qualitative behavior of the functions $T_i(\tau)$.

The presented numerical calculations are consistent with the previous analytical ones, indicating the latter's validity.

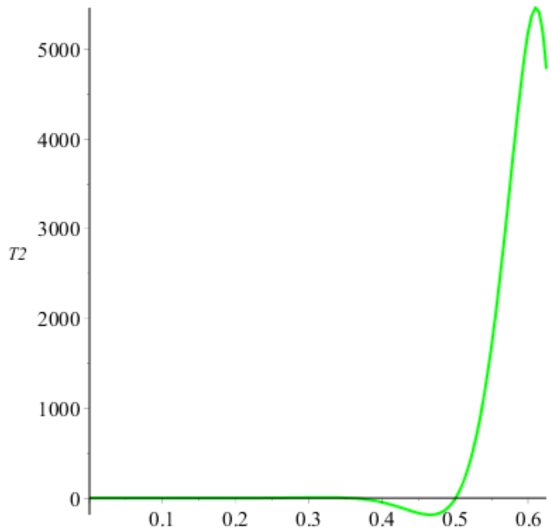

Fig. 6. Graph of the function $T_2(\tau)$ at $p = 150$

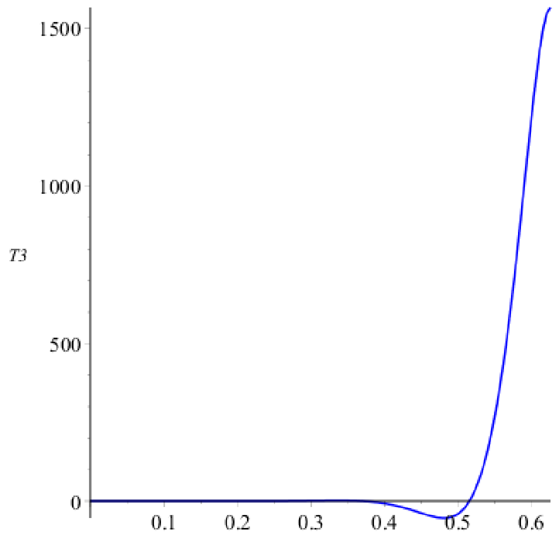

Fig. 7. Graph of the function $T_3(\tau)$ at $p = 150$

Conclusion

The problem of the stability of motion of a viscoelastic rod under the influence of a tracking force has been solved. The solution is presented as a countable series of beam functions. The obtained results are as follows:

1. The smallest number of expansion terms required to solve the problem has been analytically established to be $N = 3$.
2. The dependence of the critical load p^* on the coefficient of internal viscosity k is provided.
3. The paradox of destabilization has been confirmed: $p^*\big|_{k\to 0} << p^*\big|_{k=0}$.
4. It has been proven that the existence of any viscosity, no matter how small, leads to a finite decrease in the limit of stability.
5. Numerical studies have confirmed the analytical results presented.

References

[1] V.I. Feodos'ev, *Selected problems and questions in the strength of materials*, Mir Publisher, 1977.

[2] M. Zhravkov, Y. Lui and E. Starovoitov, *Mechanics of Solid Deformable Body*, Springer, 2023, doi: 10.1007/978-981-19-8410-5.

[3] A. Luongo, M. Ferretti and F. D'Annibale, *Paradoxes in dynamic stability of mechanical systems: investigating the causes and detecting the nonlinear behaviors*, Springerplus, 2016, doi: 10.1186/s40064-016-1684-9.

[4] A.P. Seiranyan, "Paradoks destabilizatsii v zadachakh ustoichivosti nekonservativnykh system," *Uspekhi mekhaniki*, Vol. 13, pp. 89–124, 1990.

[5] A. E. Baikov and P. S. Krasil'nikov, "The Zigler effect in a non-conservative mechanical system," *Journal of Applied Mathematics and Mechanics*, Vol. 74, Issue 1, pp. 51–60, 2010, doi: 10.1016/j.jappmathmech.2010.03.005.

[6] A. P. Filin, *Prikladnaya mekhanika tverdogo deformiremogo tela*, Moscow: Nauka, Vol. 3, 1981.

[7] S. A. Agafonov and D. V. Georgiyevskii, "The dependence of the jump in the critical follower force for a viscoelastic bar on the form of the non-linear internal viscosity," *Journal of Applied Mathematics and Mechanics*, pp. 367–373, 2011, doi: 10.1016/j.jappmathmech.2011.07.016.

[8] G. Samolyk, "Investigation of the Cold Orbital Forging Process of an AlMgSi Alloy Bevel Gear", *Journal of Materials Processing Technology*, Elsevier, No. 213, pp. 1692–1702, 2013, doi: 10.1016/j.jmatprotec.2013.03.027.

[9] A. Luongo and F. D'Annibale, "A paradigmatic minimal system to explain the Ziegler paradox," *Continuum Mechanics and Thermodynamics*, vol. 27, pp. 211–222, 2015, doi: 10.1007/s00161-014-0363-8.

[10] C. Franco and J. Collado, "Ziegler paradox and periodic coefficient differential equations," in *Proc.12th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE)*, Mexico City, Mexico, 2015, pp. 1–5, doi:10.1109/ICEEE.2015.7357933.

[11] F. D'Annibale and M. Ferretti, "On the effects of linear damping on the nonlinear Ziegler's column," *Nonlinear Dynamics*, vol. 103, pp. 3149–3164, 2021, doi: 10.1007/s11071-020-05797-y.

Дослідження стійкості руху в’язкопружного стрижня

І.А. Костюшко[1] • Г.Г. Шаповалов[1]

[1] *КПІ ім. Ігоря Сікорського, Київ, Україна*

Анотація. *Стійкість неконсервативно навантажених пружних та непружних тіл – класичний розділ механіки деформованого твердого тіла, який викликає цікавість на протязі багатьох років. В даній роботі досліджується стійкість руху вільного стрижня, на один із кінців якого діє стала по величині сила, що стежить. Задача є цікавою в практичному застосуванні, оскільки її можна розглядати як спрощену модель ракети, що рухається під дією реактивної сили. Визначним співвідношенням матеріалу стрижня є модель Кельвіна-Фойгта. Розв’язок задачі представлено у вигляді розкладу по балочним функціям. Обґрунтовано кількість доданків даного розкладу. Визначено значення критичного навантаження при існуванні та відсутності в’язкості. Встановлено, що існування ненульового значення коефіцієнта внутрішньої в’язкості у моделі Кельвіна-Фойгта призводить до суттєвого зменшення величини критичного навантаження в порівнянні із моделлю пружного стрижня. Наведені аналітичні результати підтверджуються чисельними розрахунками.*

Ключові слова: *модель Кельвіна-Фойгта, критична сила, коефіцієнт в’язкості, стійкість, балочні функції.*