

# Determination Modal parameters a Turning machining system

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**Abstract.** Among various machining processes, turning accounts for the largest volume of chip removal worldwide, so much attention is paid to predicting stability and preventing chatter, with a special emphasis on machining systems. The use of CNC machines expands the possibilities for controlling the cutting process in accordance with the Stability Lobes Diagram (SLD), which allows not only to ensure the required quality and the reduce of vibrations, but also to select the cutting mode that ensures maximum productivity. Designing a SLD involves building a mathematical model of the machining system, which is associated with determining its dynamic characteristics. A dynamic model of the machining system of a lathe has been developed as a composition of the main units using the receptance coupling method, which made it possible to calculate the rigidity of the entire system, reduced to the tip of the cutter, as a function of the longitudinal coordinate when machining a workpiece installed in a chuck and in a chuck and rear center. The cutting process model represents the components of the cutting forces acting along the coordinate axes, which allows it to be integrated into the structure of the machining system. A method for experimental modal analysis of the machining system of a lathe using a hammer, accelerometer and a storage two-channel oscilloscope has been developed. Software for determining the frequency response function using a digital file of the experimental impulse response function has been created.

**Keywords:** experimental modal analysis, dynamic of a lathe, model of turning process.

## 1. Introduction

One of the main obstacles to increasing the productivity of turning is the vibrations of the machining system, which lead to the appearance of chatter on the surface of the part, increased wear of the tool and the machine. Dynamic processes in the machining system leading to the occurrence of regenerative oscillations are the main reason for the appearance of vibrations. To eliminate such negative phenomena in the cutting processes different methods are used that can be conditionally divided into passive chatter control (PCC), active chatter control (ACC) and methods based on the use of the cutting mode with accordance to the stability lobes diagrams (SLD) [1].

The first two methods always involve some modernization of the machining system, whereas the method based

on the SLD assumes the use of the existing capabilities of any CNC machine to control all components of the cutting mode over a wide range.

The efficiency of all these methods depends entirely on the adequacy of the mathematical model of the process used in the design. The choice of parameters of dynamic vibration compensators, characteristics of the spindle speed variation, systems for introducing movements in the machining system in antiphase to regenerative vibrations, is always based on the mathematical model of the machining system and the cutting process. In the future, the design results are implemented in the corresponding structures or automatic control systems.

Despite the simplicity of using the results of SLD, it is especially important to have an adequate model of the process, since errors in assigning the cutting mode can lead to an increase in the vibration level instead of the expected decrease. The model should include the characteristics of the cutting process and the dynamics of the machining system, i.e. represent the process as a closed system with feedback on the main interactions [2]. In addition, the peculiarity of the structure of the dynamic system associated with the machine layout and the type of machining (turning,

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milling, grinding, etc.) impose mandatory conditions for creating a mathematical model.

Since the machining system is under the action of force excitation, which causes a response of the elastic system in the form of oscillations, the model of the cutting process must establish a connection between the cutting mode and the cutting force. The model of the dynamic machining system depends on the number of accepted degrees of freedom, but in any case, must be identified by stiffness, natural oscillation frequency and damping in the form of the oscillation damping coefficient. Obviously, all such parameters must be determined by experimental methods, since the success of control due to the selection of the cutting mode according to the SLD completely depends on the adequacy of the model. This article is devoted to the solution of these problems, relevant for the design of control strategies for eliminating chatter in the turning machining system.

## 2. Problem status analysis

Vibrations occur in an elastic machining system that is under the action of a force disturbance, which leads to periodic displacements of the tool and the workpiece in the cutting zone, changes in the thickness of the chip and, accordingly, the cutting force. This feedback reaction is the main property of machining systems, leading to the occurrence of regenerative oscillations. The effectiveness of all chatter elimination methods depends on the degree of adequacy of the mathematical model of the process, which includes both the cutting process itself and the dynamics of the elastic system in which it is carried out.

Among the various machining processes, turning accounts for the largest part of the chip removal volume in the machining process so much attention is paid to stability prediction and chatter prevention with a special emphasis on machining systems [3]. The proposed methods and approaches for chatter reduction and prevention are focused on mechanistic, analytical, numerical procedures for predicting stability in turning. Models of the cutting process in turning, which can be used to describe dynamic processes, should be focused on force dependencies that reflect the mechanics of machining and mechanical loads on cutting tools and workpieces [4]. The analysis pays attention to the influence of process parameters, cutting tool properties and workpiece material properties on cutting forces.

Cutting force affects the deformation of the workpiece, its dimensional accuracy, chip formation and the stability of the machining system. A direct approach to study the cutting force in machining is very expensive and time-consuming, especially when a wide range of parameters are involved: tool geometry, materials, cutting conditions, etc. An attempt is made to give a brief overview of different approaches to estimating cutting forces and the influence of cutting parameters on cutting forces in turning [5]. Cutting force prediction methods for turning are proposed using Gaussian Process Regression (GPR), Support Vector

Machines (SVM) and Artificial Neural Networks (ANN). The performance comparison between GPR, SVM and ANN shows that GPR is an effective method that can provide high accuracy in predicting cutting force in turning [6].

It is clear from the presented works that such models require statistical tests and are poorly compatible with the dynamic structure of the machining system. Therefore, for use in the general model of the dynamic behavior of the machining system, it is necessary to obtain the dependences of the cutting force components on the cutting mode, directed along the coordinate axes, since the dynamic model includes modal parameters determined by the experimental method also along the coordinate axes of the machine.

Identification of the mathematical model of the cutting process necessarily involves determining the cutting force coefficients when using a general model based on a mechanistic approach and applicable to any cutting operation [7]. The cutting force coefficients are estimated in the frequency domain with the development of digital models of all stages of part production. The intersection of the tool and the workpiece along the tool path is estimated in discrete steps, which are then used to calculate the chip area, cutting force, torque, power and energy consumed by the machine, and to detect the occurrence of vibration. The dynamics of the CNC system are included in the digital model to estimate the true tangential feed and machining cycle time [8].

Experimental modal analysis (EMA) methods are widely used to identify the dynamic parameters of the machining system [9]. Such methods involve the action of a pulse impact on the dynamic system with a special hammer or the application of a harmonic effect with a special shaker, the frequency of which can be varied [10]. The dynamics of machine tools can differ significantly from the results obtained by traditional static EMA, which leads to the need for a method of operational identification.

However, due to the lack of the ability to measure the input excitation, it is impossible to obtain the Frequency Response Function (FRF), which in the classical sense is the ratio of the output signal to the input signal as a function of frequency [11]. Methods for studying the signs of tool chatter at the initial stage during turning using a microphone are also used [12]. It is claimed that the proposed methodology will serve as a guide for researchers and machine operators to identify tool chatter at the initial stage, study its severity and, finally, suppress it by properly choosing the input parameters of turning. An SLD was built to access the stability mode, although the algorithm for building and the necessary input parameters of the machining system are not presented.

Self-excited chatter vibrations are one of the factors affecting the reduction of cutting efficiency, especially when machining highly compliant machine parts. To dampen vibrations, it is recommended to use the "correct" choice of machining parameters [13]. These parameters can be determined by analyzing the stability of the cutting

process, which requires knowledge of the dynamic properties of the machine-tool-workpiece system. Usually, these properties are determined experimentally, which is difficult in industrial practice.

This paper [14] presents a method in which the dynamic properties are calculated using a single-board computer integrated with a Computer Numerical Control (CNC) system, without the need for additional experimental tests. This is possible with the coupling re-acceptance approach, which allows obtaining the workpiece geometry by analyzing the machining program and then determining the dynamic properties of the machine-workpiece system. These properties are the input data for the presented algorithm, which facilitates the selection of cutting parameters, ensuring stable machining of highly compliant machine parts. The receptance coupling method used in this paper is a technique in structural dynamics that allows the receptances (or FRFs) of individual components to be analytically combined to predict the dynamics of the entire assembly. This method is particularly useful in modeling machines and tools where it is necessary to predict the frequency response at a specific point, such as the tip of a tool.

When analyzing new technologies aimed at studying the dynamic processes occurring during cutting, it is noted that the mathematical model should be built taking into account the closed loop of the elastic technological machining system and the function of the delay argument, representing machining along the track [16]. It is argued that it is the machining along the track, which is represented in the model by positive feedback through the function of the delay argument that is the main reason for the occurrence of regenerative oscillations.

It is proposed to identify such parameters as a result of experimental modal analysis by striking the elements of a tool and a workpiece with an impact hammer and processing the pulse signal by Fast Fourier Transform (FFT). The article [17] proposes a method for determining such dynamic parameters of the machining system as natural oscillation frequencies, oscillation damping coefficients and the stiffness of the replacement model of a single-mass system in the direction of the coordinate axes of the machine tool-CNC. A technique has been developed for adapting experimental results to the adopted mathematical model of the machining system, presented in the form of two masses, each with two degrees of freedom, according to the equivalence of the signal spectrum power or its spectral density.

Thus, all researchers of turning point to the tendency of machining systems to generate vibrations, which leads to negative consequences. The most appropriate way to combat vibrations is to assign a cutting mode in accordance with SLD. However, to design such a diagram, it is necessary to determine the dynamic characteristics of the machining system, which can be attributed to modal parameters. When simulating turning systems, there is a problem of adapting the obtained experimental results of frequency responses to the model that is adopted for designing SLD.

In addition, there is a problem of identifying stiffness, vibration damping coefficient and cutting force coefficient. The article presents the authors' experience in solving such urgent problems for turning.

### 3. The aim and objectives of the study

This work is aimed at creating a method for determining the modal parameters of a mathematical model of a turning machining system, which is based on experimental modal analysis, the results of determining the components of the cutting force, stiffness and the receptance connection method, which allows combining individual components to predict the dynamics of the entire assembly and construct SLD.

To achieve the goal, it is necessary to solve the following problems:

- constructing a structural diagram of the entire machining system based on the receptance connection method;
- determining the influence of the cutting force components in the structure of the dynamic model of the machining system on elastic movements during cutting;
- conducting an experimental modal analysis of the dynamics of the machining system and determining its frequency responses for the SLD design model.

### 4. The study materials and methods

The object of the study is the lathe machining system, and the subject of the study is the dynamic properties of the system, on the basis of which the SLD is designed, allowing to select a vibration-free cutting mode, providing the highest possible productivity at the required quality.

The cutting process is considered during its implementation in the machining system, when it is necessary to use the dynamic responses of the elastic system and the EMA results to determine its components. To perform the modal analysis, experimental studies are used using the Impact Hammer Model 086D05, the dual-beam storage oscilloscope model XDS 3202E, the accelerometer RSV 353B15, the multicomponent sensor MCS10, the amplifier ClipX BM40 and the corresponding programs for signal processing and their representation in digital format.

To process the digital signals of the modal tests, an original computer program was developed, providing a FFT and determining the natural frequency and the damping coefficient of the oscillations of the dynamic system.

### 5. Results of identification machining system for turning

#### 5.1. Model of the machining system

The machining system of the lathe can be represented as a large-scale structural diagram in Fig. 1. The system is subject to kinematic disturbances, which are deter-

mined by the commanded cutting mode ( $f_c$  is the commanded feed,  $H_c$  is the commanded depth,  $V_c$  is the commanded cutting speed). The force disturbances that arise within the system are determined by the cutting force components  $F_x$ ,  $F_y$  and  $F_z$ , acting along the coordinate axes. Each of the blocks of the structure is represented by a mathematical model, the type of which depends on the adopted degree of idealization.

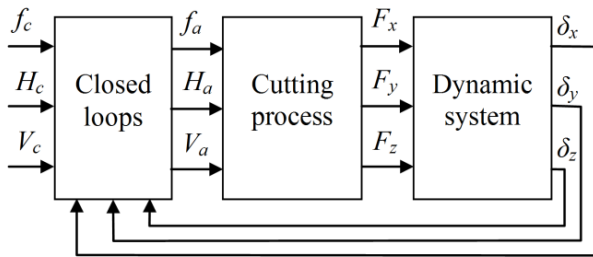


Fig. 1. Machining system structure

The closed loop block defines the most important property of the machining system, which is its closedness through internal feedback liaisons. The influence of elastic displacements of the dynamic system under the action of the cutting force components distorts the commanded components of the cutting mode, which is represented in the closed loop block by negative feedback for each coordinate. Moreover, the interaction is carried out in accordance with the physical meanings of the controlled quantities, i.e. taking into account their dimensions. In addition, the block reflects the machining along the traces, which is must be represent through the function of the delay argument by two coordinates. Thus, the following mathematical operations are performed in the block:

$$\begin{cases} f_a = f_c - \frac{d\delta_x}{dt}(1 - e^{-\tau s}) \\ H_a = H_c - \delta_y(1 - e^{-\tau s}), \\ V_a = V_c - \frac{d\delta_z}{dt} \end{cases} \quad (1)$$

where  $f_a$ ,  $H_a$ ,  $V_a$  are the actual components of the cutting mode (feed, depth, cutting speed),  $\delta_x$ ,  $\delta_y$ ,  $\delta_z$  are the elastic displacements of the dynamic system along the coordinate axes,  $\tau$  is the time of one revolution of the workpiece,  $s$  is the Laplace operator.

The cutting process during turning, according to the structure of Fig. 1, must be represented by a diagram of the interaction of the tool and the workpiece, which determines the decomposition of the cutting force  $F$  along the coordinate axes. In accordance with the mechanistic model of the cutting process, the components of the cutting force (Fig. 2) are determined by empirical dependencies on the area of the undeformed cross-section of the chip [2], [18]:

$$F_x = f_a k_f H_a^{1-m_f}, \quad F_y = f_a k_H H_a^{1-m_H}, \quad F_z = f_a H_a k, \quad (2)$$

where  $f_a$ ,  $H_a$  are the actual feed per revolution and cutting depth,  $k_f$ ,  $k_H$  are empirical coefficients,  $m_f$ ,  $m_H$  are empirical exponents,  $k$  is the specific cutting force.

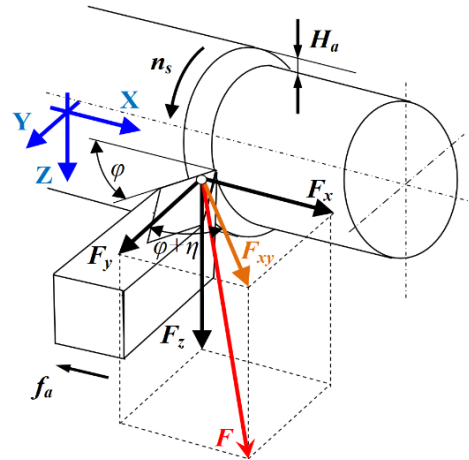


Fig. 2. Cutting force decomposition diagram

It is known that the most informative component of the cutting force is the tangential component  $F_z$  of the cutting force, which mainly determines the cutting power [18]. Therefore, in the general model of the structure of the machining system, it is proposed to determine the components  $F_x$  and  $F_y$  through the decomposition of the horizontal component  $F_{xy}$  along the coordinate axes:

$$F_x = F_{xy} \sin(\varphi + \eta), \quad F_y = F_{xy} \cos(\varphi + \eta), \quad (3)$$

where  $\varphi$  is the main angle in the cutting tool plan,  $\eta$  is the shear angle.

This scheme takes into account the deviation of the horizontal component vector of the cutting force from the normal to the cutting edge of the tool in the direction of the chip flow. The horizontal component of the cutting force depends on many factors and in the general case can be expressed through the tangential component:

$$F_{xy} = k_{xy} F_z, \quad (4)$$

where  $k_{xy}$  is an empirical coefficient, which when machining structural steels with a tensile strength of no more than 1000 N/mm<sup>2</sup>, cutting speed of 100 m/min and  $\varphi = 45^\circ$  can be in the range of  $k_{xy} = 0.55 \dots 0.65$ .

The dynamic model of the machining system of a lathe when installing a workpiece in a chuck and rear center can be represented by the diagram in Fig. 3.

In accordance with the principle of the receptance coupling method [14], [15], the dynamic system is represented as a composition of individual components to enable prediction of the frequency response function at the defining point of the entire system – at the cutter tip. When the workpiece is mounted in the chuck and the rear center, the dynamics of the system can be described by three models, each with one degree of freedom in the direction of the corresponding coordinate axis.

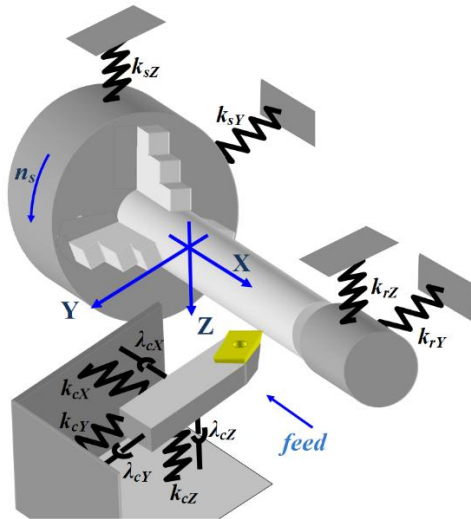


Fig. 3. Dynamic model of the machining system

All elastic movements in the machining system occur in the dynamic system, which can be represented by dynamic links described by second-order transfer functions:

$$W(s) = \frac{1/k}{\frac{s^2\delta}{\omega^2} + 2\xi\frac{s\delta}{\omega} + \delta}, \quad (5)$$

where  $k$  is the stiffness,  $\delta$  is the elastic displacement,  $\omega$  is the natural oscillation frequency,  $\xi$  is the oscillation damping coefficient. All parameters must be identified in the direction of the action of the disturbing force, which will be determined by the corresponding index of the coordinate axis designation.

Each dynamic link has its own dynamic parameters designated in the denominator of the transfer function, and the rigidity is conveniently represented in accordance with the diagram in Fig. 3 as a function of the longitudinal coordinate  $x$  as the rigidity of the entire dynamic system reduced to the tip of the cutter. This approach is determined by the purpose of the lathe machining system, since this is where the shaping of the part occurs.

When fixing the workpiece in the chuck using the classical dependencies of the theory of elasticity, one can use the formula for the reduced stiffness of the system [19] in the direction of action of the component  $F_y$  of the cutting force at the cutting point as a function of the coordinate  $x$ :

$$k_y = \frac{k_{sY}k_{cY}3EJ}{3EJ(k_{sY} + k_{cY}) + x^3k_{sY}k_{cY}}, \quad (6)$$

where  $k_{sY}$  is the spindle stiffness,  $k_{cY}$  is the tool stiffness,  $E$  is the modulus of elasticity of the workpiece material,  $J$  is the moment of inertia of the workpiece section.

For a circular section,  $J = \pi d_a^4/64$ , where  $d_a$  is the average diameter. When installing a workpiece in a chuck and a rear center, the reduced stiffness of a dynamic system is determined as for a statically indeterminate system using the method [19]:

$$k_y = \frac{l_p^2 k_{sY} k_{cY} k_{rY} 3EJ}{a_1 + a_2 + a_3 + a_4}, \quad (7)$$

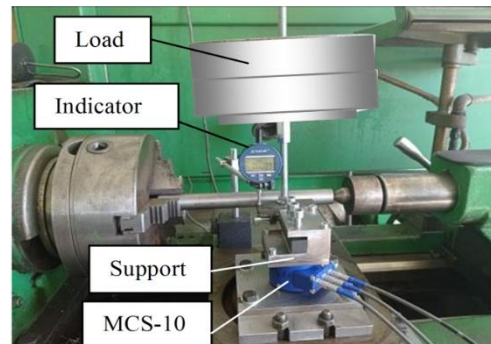
where  $a_1 = (l_p - x)^2 k_{cY} k_{rY} 3EJ$ ;  $a_2 = x^2 k_{sY} k_{cY} 3EJ$ ;  $a_3 = l_p^2 k_{sY} k_{rY} 3EJ$ ;  $a_4 = x^2 (l_p - x)^2 l_p k_{sY} k_{cY} k_{rY}$ ,  $l_p$  is the length of the part,  $k_{rY}$  is the stiffness of the rear center.

The reduced stiffness along the  $Z$  coordinate is calculated using similar dependencies with a change in indices at the corresponding stiffness in formulas (6) and (7).

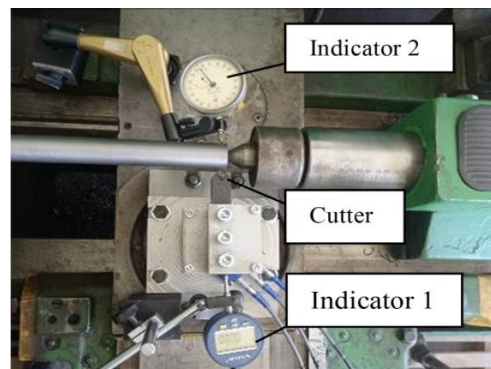
Thus, to perform further studies of the dynamics of the turning machining system, it is necessary to identify it using experimental data on determining the modal parameters [17] of the machining system: the stiffness of the dynamic links included in the system, the frequency of natural oscillations and the damping coefficients of the oscillations of the second-order links replacing them.

## 5.2. Experimental studies

To perform experimental studies, the 1A62 lathe support was upgraded to enable the measurement of cutting force components (Fig. 4). The cutter was mounted in a support secured to an MCS-10 dynamometer, each channel of which was connected to ClipX amplifiers connected to a laptop, in which the signals were processed by a special program and saved as Excel files. The dynamometer does not require calibration, which ensures that the measured signals correspond to real forces.



a

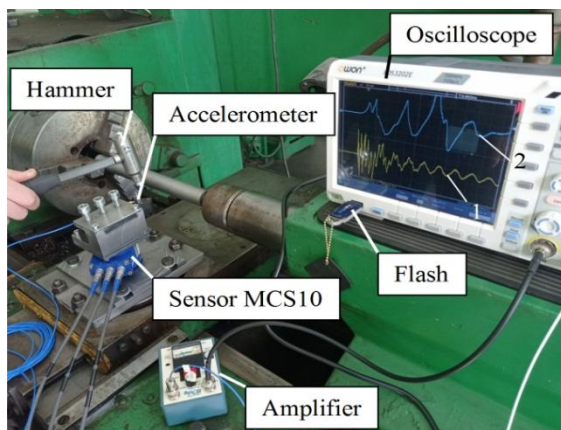


b

Fig. 4. Measurement schemes for determining rigidity: a) along the  $Z$  coordinate, b) along the  $Y$  coordinate

The measurement of the stiffness of the tool system in the direction of the Z coordinate was performed using the traditional loading scheme with disks that were installed on a special rod fixed to the cutter, and the elastic movement was recorded by an indicator. The stiffness in the direction of the Y coordinate was determined using a dynamometer. The machining system was loaded manually when the cutter contacted the workpiece in the rear center zone (Fig. 4, *b*) by moving the carriage in the direction of the Y axis. The loading (up to 1000 N) was recorded by a dynamometer and the elastic movement by indicators along the Y coordinate: cutter – indicator 1, rear center – indicator 2. Thus, with known stiffness of each component of the machining system and the dimensions of the workpiece in accordance with the scheme in Fig. 3, the stiffness at the cutter tip at any point of cutting can be calculated using formulas (6) and (7).

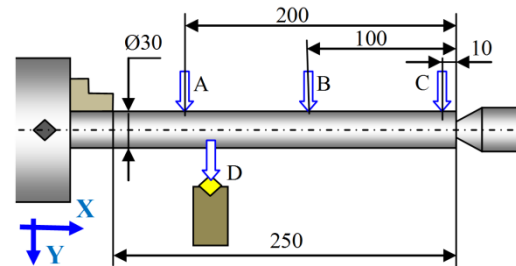
The frequency responses of each component of the machining system were determined using the experimental modal analysis (EMA) method using a model 086D05 impact hammer and an RSV 353B15 accelerometer with an RBC 480E09 amplifier, and an NI USB-9215 ADC (all from Piezotronics, Inc., USA) (Fig. 5). The signals are recorded by an Owon XDS3202E dual-channel storage oscilloscope and can be saved to flash memory as Excel files.



**Fig. 5.** Experimental setup for determining the IRF of the “cutter” system

The accelerometer is connected via an amplifier to the oscilloscope input, an impact hammer is connected to the other input, and the impact is produced in the direction of one of the coordinate axes. The result of recording the received signals in the form of IRF is displayed on the oscilloscope screen as oscillograms: the accelerometer signal is indicated by line 1, the impact hammer signal is indicated by line 2 in Fig. 5. To determine the IRF in the direction of another axis, the accelerometer location and the impact direction are changed. Similar experiments were carried out with other components of the dynamic model of the machining system: the spindle and the rear center. Measurements were performed in the machining system with the workpiece installed in accordance with the diag-

ram in Fig. 6, where the measurement locations corresponding to the greatest information content of the dynamic system are indicated by arrows A, B, C and D.



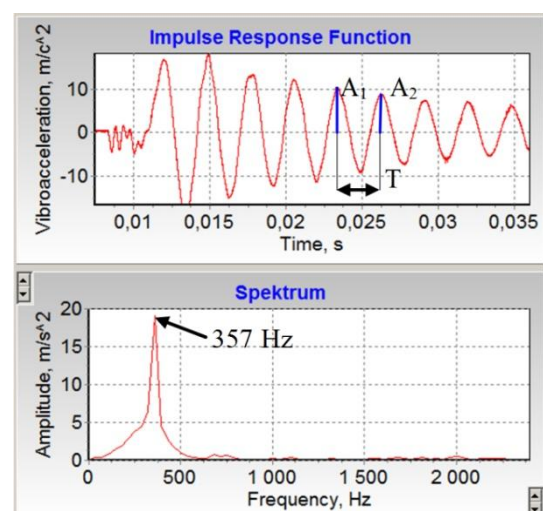
**Fig. 6.** Measurement diagram

To identify the frequency response functions (FRF) of the system components by their IRF, an application program performing Fast Fourier Transforms (FFT) was used. Fig. 7 shows the results of the experimental modal analysis of the frequency responses of the tool support in the Y-axis direction. The obtained spectrum indicates the prevalence of one main harmonic (357 Hz), and the oscillation damping coefficient  $\xi$  can be calculated from the IRF [16]:

$$\xi = \frac{\ln(A_1 / A_2)}{T\omega}, \quad (8)$$

where  $A_1$ ,  $A_2$ ,  $T$  are the amplitudes and period of oscillations on the IRF graph,  $\omega$  is the frequency of the main harmonic ( $\omega = 2\pi \times 357$  rad/s).

The obtained data, together with the experimental data on determining the stiffness in the direction of the Y axis, completely identify the oscillatory link representing the cutter system in the mathematical model of the entire machining system.



**Fig. 7.** IRF and FRF cutter system in direction of axis Y

The analysis of the obtained results shows that the dynamic model of the support in the direction of the Y axis

agrees well with the theoretical model of the oscillatory link. The reaction in the direction of the Z axis (Fig. 8) for a similar representation in the mathematical model of the entire system in the form of an oscillatory link requires the use of the approach presented in [17].

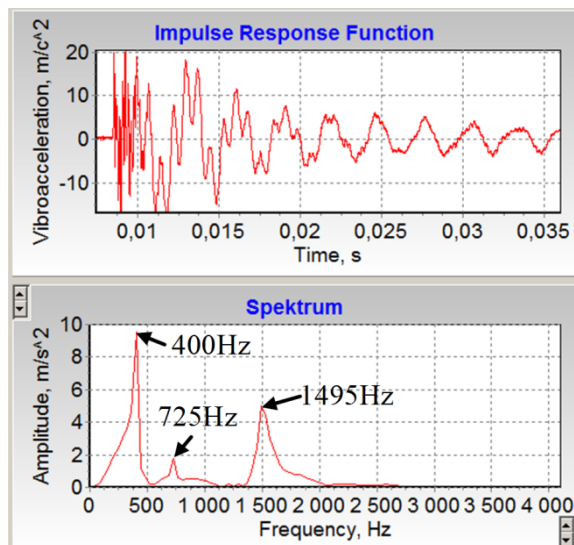


Fig. 8. IRF and FRF cutter system in direction of axis Z

The results of the experimental modal analysis of the dynamic machining system at the measurement points in accordance with the scheme in Fig. 6 are presented in Table 1. For the tool support, the measurement results in two directions are shown, and at other measurement points, such results coincide. It should be noted that during the experiments, several characteristics of the system were obtained at the indicated locations, and the table shows the averaged results for the frequencies of the main harmonics. The first harmonics with the maximum value of the spectrum amplitude are highlighted in the table.

Table 1. Experimental results

Point	Stiffness, N/mm	Frequency, Hz		
		I	II	III
A	14285	<b>235</b>	550	1650
B	5750	<b>150</b>	653	1340
C	6250	<b>240</b>	560	1180
D (Y)	12190	<b>357</b>	–	–
D (Z)	21730	<b>400</b>	725	1495

## 6. Discussion on modal analysis result

The use of CNC machines expands the possibilities for controlling the cutting process in accordance with the SLD, which allows not only to ensure the required quality and the absence of chatter, but also to select the cutting mode that ensures maximum productivity. Solutions and

algorithms for automatic diagram construction are proposed, and the control efficiency is experimentally verified [20]. However, designing a stability diagram involves constructing a mathematical model of the machining system, which is associated with determining its dynamic responses. Mathematical models are presented as a combination of dynamic elements, usually of the second order, reflecting the closed nature of the entire system with negative feedback on elastic shear and positive feedback on the delay argument for the cutting depth and feed. An model of elastic system usually consists of single-mass dynamic systems interconnected with each other, and the cutting process itself is described by a mechanistic model [2], [4]. Such characteristics must be determined by experimental methods, during experimental modal analysis, which guarantees the adequacy of the model.

Since the model of the machining system is presented in accordance with the principle of the receptance coupling method [14], as a composition of three main units of the system – a support, a spindle, a rear center – then the modal analysis was performed for each mass separately. The experiment is carried out using an impact hammer and an accelerometer attached to the studied system, and the impulse response was recorded on a storage oscilloscope. As the practice of performing such an analysis has shown, the greatest difficulty is identifying the natural frequency of oscillations of replacing equivalent models of second-order oscillatory elements [17].

The model of the cutting process is built in such a way that, using a mechanistic approach to determining the cutting force, the possibility of embedding the components of the cutting force in the structure of the entire dynamic system is realized. This approach will allow us to build a mathematical model necessary for solving the main control problem – designing a stability diagram.

The determination of the stiffness of each unit of the machining system is carried out experimentally separately, and the results are summarized in a single dependence of the stiffness at the tip of the cutter, which will allow us to predict the change in rigidity along the length of the work-piece.

The authors see the development of the methodology for determining the modal parameters of a dynamic system outlined in the article in the use of operational modal analysis [10], as well as in the creation of a special program for the automatic determination in real time of all the dynamic parameters of the machining system necessary for modeling and automatic correction of the cutting mode in accordance with the SLD.

## 7. Conclusion

1. A dynamic model of the machining system of a lathe has been developed as a composition of the main units using the receptance connection method, which made it possible to calculate the stiffness of the entire system, reduced to the tip of the cutter, as a function of the longitude-

nal coordinate when machining a workpiece installed in a chuck and in a chuck and rear center.

2. It is proposed to adapt the cutting process model using a mechanistic approach to the structure of the entire dynamic model of the machining system. The adaptation consists in decomposing the horizontal component vector, which is calculated by the introduced empirical coefficient through the tangential component of the cutting force, into components along the coordinate axes.

3. A technique has been developed for performing an experimental modal analysis of the machining system of a lathe using an impact hammer, an accelerometer, and a storage dual-channel oscilloscope. A special program has been created to process the experimental results, which makes it possible to determine the frequency response of

the tested unit. Moreover, the possibility of performing FFT during operational analysis directly during the turning operation is taken into account. This possibility is based on the speed of the embedded algorithm of the computational process.

#### Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, including financial, personal, authorship, or any other nature that could affect the research and its results presented in this article.

#### Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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## Визначення модальних параметрів токарної обробної системи

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**Анотація.** Серед різних процесів обробки, токарне оброблення становить найбільший обсяг виготовлення деталей шляхом видалення стружки у світі, тому велика увага приділяється прогнозуванню стабільності та запобіганню вібраціям, з особливим акцентом на динаміку обробної системи. Використання верстатів з ЧПК розширює можливості керування процесом різання відповідно до діаграми стабільності (ДС), що дозволяє не тільки забезпечити необхідну якість та зменшення вібрацій, але й вибрати режим різання, що гарантує максимальну продуктивність. Проектування ДС передбачає побудову математичної моделі обробної системи, яка пов'язана з визначенням її динамічних характеристик. Розроблено динамічну модель обробної системи токарного верстата як композицію основних вузлів з використанням методу рецептивного зв'язку, що дозволило розрахувати жорсткість всієї системи, зведеної до вершини різця, як функцію поздовжньої координати при обробці заготовки, встановленої в патроні, а також у патроні та задньому центрі. Модель процесу різання відображає складові сил різання, що діють вздовж координатних осей, що дозволяє інтегрувати її в структуру обробної системи. Розроблено метод експериментального модального аналізу обробної системи токарного верстата з використанням ударного молотка, акселерометра та двоканального запам'ятовуючого осцилографа. Створено програмне забезпечення для визначення частотної характеристики за допомогою цифрового файлу експериментальної імпульсної характеристики.

**Ключові слова:** експериментальний модальний аналіз, динаміка токарного верстата, модель процесу точіння.

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