

# Inverse Problems and Their Use for the Design of Aviation Navigation Equipment

Volodymyr Povhorodnii<sup>1,2</sup> - ORCID <https://orcid.org/0000-0003-1640-1004>

Received: 19 January 2026 / Revised: 9 April 2026 / Accepted: 18 April 2026

**Abstract.** The article proposes a method for determining the maximum thermal load from the temperature (thermal) stress measured with a certain error by solving the inverse problem of thermal conductivity and thermoelasticity. The determination of the maximum thermal load in the same way as the regulation of external and internal temperature and power loads, at which the temperature stresses or displacements in the structural elements within acceptable limits are achieved, are of significant theoretical value and are of great practical value, namely for non-destructive testing tasks. An expedient way of finding these quantities as a function of time and geometric coordinates is to solve the inverse problems of thermal conductivity and thermoelasticity, i.e., to determine the temperature field based on the field of temperature stresses. To obtain a stable solution to the inverse problem of thermoelasticity, Tikhonov's method is used with an effective search for the regularization parameter. Tikhonov's functional reflects the deviation of the temperature stress obtained as a result of observation from that calculated on the basis of an approximate solution of the direct problem of thermoelasticity by the finite and boundary element's methods. In this functional, the stabilizing functional with the regularization parameter is used as the term to the square of the indicated deviation. The search for the regularization parameter is carried out using an algorithm similar to the search algorithm for the root of a nonlinear equation. The use of influence functions in the method allows one to represent temperature and temperature stress depending on the same vector, which greatly facilitates the implementation of the iterative process. The proposed method allows, without bringing the research object to failure, to determine the load at which it will be destroyed i.e. this problem of non-destructive task. The effectiveness of this method lies in the fact that its application reduces the cost of complex experimental studies of objects and eliminates the need to create analytical methods that accompany these studies.

**Keywords:** inverse problem, thermal stress, functions of influence, spline, identification, regularization.

## 1. Introduction and history of inverse problems

As is known, many mathematical concepts and problem statements arose as a result of studying certain physical processes or phenomena. This is especially true for the theory of inverse and ill-posed problems. Plato's philosophical assertion that humanity in the process of cognition has access only to shadows on the wall of a cave and echoes (data from the inverse problem) was a harbinger of Aristotle's solution to the problem of reconstructing the shape of

the Earth from its shadow on the Moon. The introduction of the physical concept of instantaneous velocity led I. Newton to the discovery of the derivative, and the problem of instability (incorrectness) of the problem of numerical differentiation of a function given approximately is still relevant today. Lord Rayleigh's research on acoustics prompted him to raise the question of the possibility of finding the density of an inhomogeneous string from its sound (the inverse problem of acoustics), which anticipated the development of seismic exploration, on the one hand, and the development of the theory of spectral inverse problems, on the other. The study of the motion of celestial bodies and the problem of estimating unknown quantities based on measurement results containing random errors led A. Legendre and K. Gauss to overdetermined systems of algebraic equations and to the creation of the least squares method. O. Cauchy proposed the steepest descent method for finding the minimum of a function of several variables. L. Kantorovich generalized, developed and applied these ideas to operator equations in Hilbert spaces. Currently, the

<sup>1</sup> State University "Kiev aviation institute", Kyiv, Ukraine, <https://ror.org/01f74x078>

<sup>2</sup> National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Kyiv, Ukraine, <https://ror.org/00syn5v21>

✉ V. O. Povhorodnii  
povgorodnij@gmail.com



steepest descent method, along with the conjugate gradient method, are among the most popular in solving ill-posed problems. It is worth noting that L. Kantorovich was the first to draw attention to the fact that the method he proposed converges only in terms of the functional in the case where the problem is ill-posed.

Thus, individual inverse and ill-posed problems have long been the object of research in various fields of knowledge. However, the mathematical features of ill-posed problems were formulated by Hadamard only at the beginning of the 20th century, and at the same time the question arose about the advisability of finding a unified approach to solving such problems. The thesis that there are no ill-posed problems, but rather poorly posed problems, cooled some researchers, while encouraging others to seek new ways to solve these “incorrect” problems. R. Courant’s conviction that unstable problems have no physical meaning did not prevent him from solving the highly ill-posed problem of restoring a function from its spherical means.

S. Sobolev was a scientific consultant to V. K. Ivanov on his doctoral dissertation “Research on the inverse problem of potential theory”, which provided a theoretical basis for a number of inverse problems of gravity exploration.

It follows from the classical Cauchy-Kovalevskaya theorem that the solution of a wide range of inverse and ill-posed problems exists and is unique, but only in the class of analytic functions. L. Ovsyannikov proved that the requirement of analyticity with respect to the output variable can be significantly weakened. V. G. Romanov, developing the method of scales of Banach spaces. L. Ovsyannikov and L. Nirenberg showed that for a wide range of inverse problems it is possible to get rid of the condition of analyticity with respect to two variables – with respect to the output spatial variable and with respect to the time variable.

These studies opened the way to the study of multi-dimensional inverse problems of geophysics, the basic model of which is a horizontally layered medium. It is impossible to cover all aspects of the theory of inverse problems and its applications in one article.

The inverse scattering method was applied to solve nonlinear equations of mathematical physics (Korteweg-de Vries equation, nonlinear Schrödinger equation, Kadomtsev-Petviashvili equation, etc.) and stimulated new research in various fields of mathematics and physics (spectral theory of differential operators, classical algebraic geometry, relativistic strings, etc.).

The inverse scattering problem method is called the pearl of mathematical physics of the 20th century. The results of A. Alekseev and S. Goldin on the application of the spectral theory of inverse problems and integral geometry in geophysics became the theoretical basis for many geophysical methods (inverse kinematic and dynamic problems of seismics).

It should be noted that the recognized successes of the current generation of Siberian geophysicists are largely determined by their high mathematical training at the geological and geophysical faculty of NSU. The author of the

article was fortunate to work at the inverse problem department in those years, when a creative union of teachers of physicists Vigak, A. Yasinski, O. Alifanov [1], A. Tikhonov, S. Goldin, L. Tabarovskiy, M. Epov, Yu. Dashevskiy, etc. and mathematicians (M. Lavrentyev, A. Alekseev, V. Romanov, T. Godunova) was created there.

Discussions about what kind of mathematics and how much should be taught to geophysicists were regular occurrences at faculty meetings, and the debates often resembled discussions at scientific conferences.

The list of examples could be continued, but we will only note that the world-renowned founders of the theory of ill-posed problems are A. N. Tikhonov, V. Ivanov and M. Lavrentiev. The foundations of the theory of inverse and ill-posed problems were laid in the works of these scientists.

One of the main ideas was that when studying ill-posed problems it is necessary to narrow the class of possible solutions. The most important role is played by the choice of the set in which an approximate solution is sought (the correctness set).

Most often, this set is chosen to be compact, which makes it possible to justify the convergence of regularizing algorithms, helps to choose the regularization parameter, and evaluate the deviation of the approximate solution from the exact solution of an ill-posed problem. The results of mathematical research were applied to solve a number of specific inverse problems in geophysics, radar, astronomy, and medical tomography.

From the end of the twentieth century to the present day, mathematics and all natural sciences have seen an unprecedented growth of interest in inverse and ill-posed problems.

## 2. Literature review

Scientific results in this area by A. N. Tikhonov and V. K. Ivanov, and later M. M. Lavrentiev, Yu. Anikonov, V. Kireyotov, V. Romanov and S. P. Shishatsky, Vigak, A. Yasinski, N. Guk.

Adams and Burt [2] studied the thermoelastic vibrations of a multilayer rectangular plate subjected to thermal shock. Dang, Khobragade, and Durge [3] studied the three-dimensional inverse thermoelasticity problem for a thin rectangular plate. Gryza [4] and Hematyan [5], solving one-dimensional inverse thermoelasticity problems, determined the heating temperature and heat flux on the surface of an infinite isotropic plate. Lamba and Khobragade [6] investigated the thermoelasticity of a thin rectangular plate. Ishihara and Noda [7] conducted a theoretical analysis of the thermoelastic deformation of a circular plate under partially distributed heat supply. Jadhav and Khobragade [8] investigated thermoelasticity for a thin finite rectangular plate with an internal heat source. Kozlov V. and other authors provided a solution to the inverse thermoelasticity problem [9]. Khobragade, Deshmukh [10] proposed a solution to the inverse thermoelasticity problem for a circular plate. Deshmukh, Lamba and Khobragade [11] proposed a

solution to the inverse thermoelasticity problem for a rectangular plate. Gume and Khobragade [12] investigated the deflection of a thick rectangular plate. Roy, Bagade, and Khobragade [13] investigated the distribution of thermal stresses in a rectangular beam, Roy and Khobragade [14] solved the thermoelastic problem for an infinite rectangular plate. Singru and Khobragade [15]–[17] investigated the analysis of thermal stresses in a thin rectangular plate. Sutar and Khobragade [18] solved the inverse thermoelasticity conversion with internal heat release in a thin rectangular plate. Tanigawa and Komatsubara [19], conducted an analysis of thermal stresses in a rectangular plate and determined the coefficient of intensity of compressive stresses.

Consideration of the thermal regime of equipment with a gradually increasing degree of detail from the largest bodies (complexes, blocks, etc.) to individual elements of electrical circuits [20]. In [21], [22], an analytical solution to the thermal and mechanical design problem is presented, at the early stages of equipment development.

As a review of the literature has shown, a distinctive feature of the thermal regime of cassette-type blocks is the unevenness of temperature fields both in the plane of the boards and in the direction perpendicular to the boards (with uniform heat dissipation on the boards).

Currently, the permissible temperature stresses of fiberglass are little known, and their measurement entails great technical difficulties. Therefore, it is proposed to use the inverse problem of the thermoelasticity solution when, based on the permissible temperature stress, it is necessary to determine the permissible limits of change in the initial thermal parameters (temperature and heat flux).

### 3. Research methodology

**The object of research** or, in our case, the thermal object is understood to mean both technical devices (structures, devices, machines and their elements) and thermophysical processes occurring in thermodynamic systems, which include these devices.

**Identification** – building a model of the object under study or determining some of its characteristics based on the analysis of input and output data of the object. Two types of identification are usually considered: identification in a broad sense (structural identification) – determination of the structure of the model from physical considerations based on any available information about the object research, and identification in the narrow sense (parametric identification) – assessment of parameters that determine the given (known) behavior of an object with a known structure of the model.

However, in both of these definitions, generally speaking, which are comprehensive in terms of content, there is no sign explaining why the word “identification” was chosen to denote the operations (actions, processes) mentioned above.

Apparently, the identification process is connected with matching the model of the phenomenon (object) with the physical phenomenon (object) itself, i.e. in the case of,

for example, parametric identification, it is about determining such parameters of the thermal and mechanical object's model that would make it identical to the considered thermal and mechanical objects.

In particular, the identification of the coefficients of the boundary conditions means the determination of such coefficients at which the heat flow will be identical to the actual heat flow at the boundary between a solid body and a liquid (gas).

The identification of thermophysical characteristics corresponds to the determination of such coefficients of the heat conduction equation, which make the properties of the model identical to the thermophysical mechanical properties of the object, etc.

**Structural identification** (or identification in a broad sense) is the definition of such a model structure that makes the model's response identical to the response of a thermal object under similar input effects.

**Parametric identification** (or identification in the narrow sense) is the determination of the parameters of a model (with its known structure) that make the model's behavior identical to the behavior of a thermal object. The results of measurements of some properties of the system, its linearity or nonlinearity, discreteness or continuity, etc.

**Statistical identification** is an identification that is usually carried out in real problems, where the input values are almost always random. To carry out the identification procedure, it is necessary to have some a priori and a posteriori information about the system or object being studied. A priori information includes information about dynamic and stochastic properties; the lot is described in the general case by a nonlinear heat conduction equation.

The transformation of the mathematical model of the heat conduction process is carried out in order to bring it to a form more convenient for computational implementation. The transformations traditionally used in modeling include, first of all, the translation of the mathematical model from one coordinate system to another.

Less commonly used is the conformal transformation of a flat region. In the case of linear thermal processes, various integral, differential or linear functional transformations are used.

When solving nonlinear problems, transformations are used that fully or partially linearize the original mathematical model. Such transformations include Kirchhoff, Goodman, Schneider, Boltzmann, and other substitutions.

It makes it possible to get rid of nonlinearity in the left-hand side of the heat conduction equation. True, the boundary conditions equations sometimes become more complicated due to this operation, but the volume of calculations is significantly reduced and the structure of the modeling tools is simplified.

**Mathematical model of the process of thermal stress generation caused by the inhomogeneity of the temperature field.**

Temperature distribution  $T$  in space and time in solid isotropic bodies in the absence of internal sources or sinks of heat. It makes it possible to get rid of the nonlinearity on the left side of the heat conduction equation. Temperature

distribution  $T$  in space and time in solid isotropic bodies in the absence of internal sources or sinks of heat is described in the general case by a nonlinear heat conduction equation.

If the thermophysical characteristics of the body under study depend weakly on temperature and can be considered constant, then the heat conduction equation turns into a linear equation – the Fourier equation. In the mathematical model we use functions of influence.

Now our case. We have the equation:

$$\frac{d}{dr} \left( \frac{1}{r} \frac{d(ru)}{dr} \right) = (1+\nu)\alpha \frac{dT}{dt}, \quad a < r < b, \quad (1)$$

and boundary conditions:

$$\begin{aligned} \frac{E}{1-\nu^2} \left[ \frac{du}{dr} + \nu \frac{u}{r} - (1+\nu)\alpha T \right]_{r=a} &= 0, \\ \frac{E}{1-\nu^2} \left[ \frac{du}{dr} + \nu \frac{u}{r} - (1+\nu)\alpha T \right]_{r=b} &= 0. \end{aligned} \quad (2)$$

The boundary value problem (1)–(2) is linear, which means model of the process of thermal stress generation caused by the inhomogeneity of the temperature field. Let us represent the solution as a sum:

$$\frac{dW}{dr}(a) + \frac{\nu}{a}W(a) = (1+\nu)\alpha T(a). \quad (3)$$

Here, in addition to the linear combination of solutions  $\sum_{i=1}^{n+1} T_i U_i(r)$ , there is a term in the sum (3) and this is  $W(r)$ . We will also decompose the temperature into the same number of components and this will be the solution to the boundary value problem of heat conductivity:

$$T = W(r) \sum_{i=1}^{n+1} T_i \varphi_i(r). \quad (4)$$

Now we substitute the differential equation and boundary conditions (1)–(2) into the expression for the expansion of solutions (3) and (4) of the boundary value problem. We obtain solution to the inverse problem of the thermoelasticity we select from the boundary value problem (4)–(5) the boundary value problem for the function  $W(r)$ :

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d(rW)}{dr} \right] = 0, \quad (5)$$

$$\frac{dW}{dr}(a) + \frac{\nu}{a}W(a) = (1+\nu)\alpha T(a), \quad (6)$$

$$\frac{dW}{dr}(b) + \frac{\nu}{b}W(b) = (1+\nu)\alpha T(b), \quad (7)$$

and boundary value problems for functions  $U_i(r)$  (influence functions):

$$\frac{d}{dr} \left[ \frac{1}{r} \frac{d(rU_i)}{dr} \right] = (1+\nu)\alpha \frac{d\varphi_i}{dt}, \quad a < r < b, \quad (8)$$

$$\frac{dU_i}{dr}(a) + \frac{\nu}{a}U_i(a) = 0, \quad (9)$$

$$\frac{dU_i}{dr}(b) + \frac{\nu}{b}U_i(b) = 0. \quad (10)$$

Finding the regularization parameter.

What does the quantity represent:

$$I = \sqrt{\int_a^b \left[ \sigma_r(r) + \sigma_r^{ex} \right]^2 r dr}, \quad (11)$$

$\sigma_r^{ex}$  – measured radial stress and  $\sigma_r(r)$  – is the calculated radial stress. These are two different functions. If we take their difference, square it, and integrate it over the area of their variation, we get the square of the deviation from.

If we integrate not the square of the deviation (difference), but the differences themselves, then the deviations at different points may have different signs and when integrating, the deviations at all points will not be summed up, some will be subtracted. In this case, the deviations at different points will cancel each other out, but it is necessary that the deviations at each point be summed up.

Then we take the square root to compensate for the square under the integral. It turns out to be a sum of all deviations. We can also divide by the measure of the integration area  $b-a$  and then we get the average deviation for all points. But we did not do this here, because it is not essential. This is what this  $I$  is.

The idea of finding the regularization parameter is as follows. When changing  $\xi$  we obtain  $\sigma_r(r)$  and calculate the deviation  $I$ . But this is not enough, because the true value of this deviation is unknown, since  $\sigma_r^{ex}$  it has an error. Then we will do this: we will find the interval (numerical segment) in which the true deviation can be located.

Then we take the square root to compensate for the square under the integral. It turns out to be a sum of all deviations. We can also divide by the measure of the integration area and then we get the average deviation for all points. But we did not do this here, because it is not essential. This is what this  $I$  is.

Let us calculate the integral (6) using the mean value theorem for the integral and the nature of the error:

$$\sigma_{rk}^{ex} = \sigma_r(r_k) + \delta \nu_k \sigma_{r,avg}, \quad (12)$$

here  $\sigma_r(r_k)$ ,  $\sigma_{r,avg}$  – are the true stress and its average value and they are unknown,  $\nu_k$  – is a random variable with zero mathematical expectation, and  $\delta$  – is the measurement error.

Value  $I$  the integral the stresses  $\sigma(r)$  have been reduced, the first is our calculated one from (11), and the second is the true one from (12). They do not coincide, but in the limit they should be close. We find the value (unknown)  $\sigma_{r,avg}$  by what we have, namely, by the measured radial stress: for example, we sum up all the values  $\sigma_{rk}^{ex}$  and divide the sum by their number  $I_{avg}$ .

Now it is clear that the true value of the integral  $I$  must be somewhere close to the value  $I_{avg}$ , or less than it or more than it. So, we will decrease and increase by some value and then we will get the interval of change of  $I$ , where its true value is supposedly located. For example, like this  $\min I_{avg} = 0,99I_{avg}$ ,  $\max I_{avg} = 1,01I_{avg}$ .

When changing the regularization parameter (using the dichotomy method as we have, or it can be done differently), we get the radial value and integral (6) each time. We check whether it falls within the segment  $\min I_{avg} = 0,99I_{avg}$ ,  $\max I_{avg} = 1,01I_{avg}$  we have chosen or not. If it does, then we end the variation of the regularization parameter; if it does not, then we change it.

It is possible to not get into the selected segment at all when changing the regularization parameter; it may be too narrow, and there is no solution there, then you need to choose another segment, for example by increasing it.

For the calculation, we will take a board used in flight and navigation equipment. That is, the shape is round (often for components installed in a limited cylindrical space). Material: FR-4 fiberglass (typical for multilayer boards). Number of layers: 4 for complex digital-analog systems.

Mounting method: surface mount and through hole. This is an example of a commercial circuit board, which is also often used in communication devices, gyroscopes, and inertial modules. It is important that the temperature stresses do not exceed the maximum stresses in the board material (FR-4 glass-fiberboard). Otherwise, this may lead to cracks, which ultimately leads to failures (namely, to erroneous readings of flight and navigation instruments) and the creation of an emergency situation.

#### 4. Discussion of results

On the basis of complex ANSYS a program for numerical calculations is developed. The first and easiest option is based on splitting the contour of the border by a broken line. Sections of this line are associated with BE. Within BE the approximation of compensating loads seems to be piecewise linear. With these provisions, the testing of programs with simple areas for which there are known solutions was performed. In addition, at this stage, mechanical BC could be realized hinged stop and pinching. When solving a test task with a circular plate, which was pinched in the contour, it was noted the difference of 10–12 % by the value of the contour bending moment in comparison with the analytical solution. This error in the value of the contour bending moment is a consequence of replacing the contour line with a broken line. That is, it is important to take into account the curvature of the contour and, if in the quality BE are presented not straight, but corresponding arcs, the numerical solution practically coincides with the analytic. The next important point is the rate of convergence of the solution, depending on the quantity BE. Of course, the accuracy of the results obtained depends on the given accuracy of the calculation, as well as from the defi-

nition of compensating loads. It is determined that within the framework of the piecewise linear approximation, the accuracy of the solution depends directly on the increase in the number BE. The practical reception here is to partition the boundaries of a certain number of identical BE. For example, in a rectangular plate, each side is divided into a number of equal lengths BE. The accuracy of the solution at internal points is increasing rapidly with increasing the number of such BE. But when approaching the contour point, the rate of convergence of the solution slows down. This indicates that it is necessary to improve the quality of the approximation of compensating loads. With an increase in the number BE the quality situation in satisfaction should improve BC because the number of points of collocations is increased. But the increase in the number of points of collocation leads to an increase in the order of the system of linear equations and, accordingly, increases the rounding errors in its solution.

Test tasks confirm that starting from a certain point, the solution in the internal points gets worse, further increase in quantity BE becomes meaningless. Partial improving the solution can be due to uneven distribution BE on the contour (boundaries). For example, for rectangular plates, the quality of the solution improves with a relative decrease in length BE to the corner points.

For the calculation, we will take a board used in flight and navigation equipment. That is, the shape is round (often for nodes installed in a limited cylindrical space). Material: FR-4 fiberglass (typical for multilayer boards). Number of layers: 4 for complex digital-analog systems. Mounting method: surface mount and through holes. This is an example of a commercial circuit board, which is also often used in communication devices, gyroscopes and inertial modules.

It is important that the temperature stresses do not exceed the maximum stresses in the board material (FR-4 fiberglass). Otherwise, this may lead to the appearance of cracks, which ultimately leads to failures (namely, to erroneous readings of flight and navigation instruments) and the creation of an emergency situation.

The maximum value of temperature stresses is less than for fiberglass (FR-4) board material. That is, the inverse thermoelasticity problems (and, namely, their solution) can be used in non-destructive testing (Fig. 1, 2).

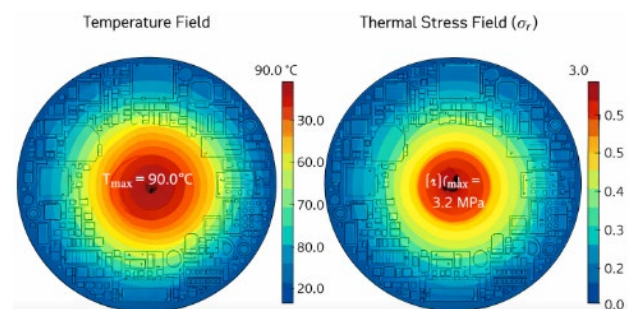


Fig. 1. Temperature field and thermal stress in the round plate

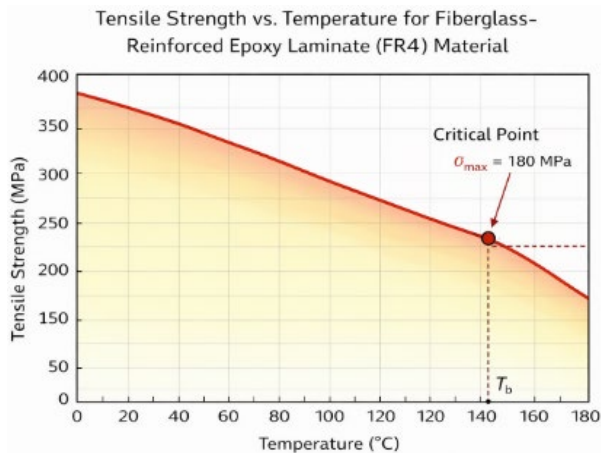


Fig. 2. Determination of the critical point for FR-4 glass-frame

## 5. Conclusions

The main disadvantage of all destructive methods is that, while allowing to estimate the magnitude of mechanical stresses with an error, they do not provide an opportunity to determine the nature of the deformations caused by these stresses that actually exist in the material, that is, to determine the state of the material and to assess how close this state is to the critical (destruction). That is, there is no sufficiently convincing expert method for assessing the correctness of determining the characteristics of the stress-strain state by physical methods that are non-destructive. Therefore, the use of solving inverse thermoelasticity problems as non-destructive control methods is an urgent issue.

Currently, for example, temperature control on surfaces of materials inaccessible to direct temperature measurement in power engineering, aircraft engineering (includ-

ing instrument making) is based on solving direct and inverse thermoelasticity problems.

For inverse thermoelasticity problems, a general universal approach to their solution has been developed, which is based on an iterative process of minimizing the Tikhonov functional, at each iteration of which this functional is represented in a quadratic form with respect to the identified parameters.

The advantages of the proposed approach to solving inverse thermoelasticity problems include the possibility of using experimental information from both one and several sensors; its adaptability to inhomogeneous environments.

A thorough analysis of the results of the multifactorial experiment allows us to state that the proposed approach allows, without bringing the object of study to destruction, to determine the load at which it will be destroyed. This is especially important for navigation equipment in unmanned aviation and in power engineering.

The economic feasibility of approaches with the solution of inverse thermoelasticity problems of the method is that it reduces the cost of complex experimental studies to find the point at which destruction can occur at maximum thermal stress in the structure. Thus, the use of inverse thermoelasticity problems is an innovative approach to achieving maximum economic efficiency in the study of the integrity of structures.

### Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, including financial, personal, authorship, or any other nature that could affect the research and its results presented in this article.

### Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

## References

- [1] O. M. Alifanov, "Inverse heat transfer problems," Berlin: Spinger, 1994, doi: <https://doi.org/10.1007/978-3-642-76436-3>.
- [2] R. J. Adams and C. W. Bert, "Thermoelastic vibrations of a laminated rectangular plate subjected to a thermal shock", *Journal of Thermal Stresses*, Vol.22, pp. 875–895, 1999, doi: <https://doi.org/10.1080/014957399280607>.
- [3] W. K. Dange, N.W. Khobragade and M. H. Durge, "Three dimensional inverse transient thermoelastic problem of a thin rectangular plate," *Int. J. of Appl. Maths*, Vol. 23, No. 2, 207–222, 2010.
- [4] A. Maciag and K. Grysa, "Treffitz Method of Solving a 1D Coupled Thermoelasticity Problem for One- and Two-Layered Media," *Energies* 2021, Vol. 14(12), 3637, 2021, doi: <https://doi.org/10.3390/en14123637>.
- [5] M. R. Hematiyan and M. Khandaei, "Inverse estimation of heat flux by strain measurement in transient thermoelasticity," *Thermal Stresses: Proceedings of the Seventh International Congress (TS 2007, Taipei, Taiwan, 4–7 June, 2007)*, Taipei: National Taiwan University of Science and Technology, Vol. 2, pp. 627–630, 2007.
- [6] H. Roy and N. Khobragade, "Transient thermoelastic problem of an infinite rectangular slab," *Int. Journal of Latest Trends in Maths*, Vol. 2, No. 1, pp. 37–43, 2012.
- [7] M. Ishihara and N. Noda, "Theoretical analysis of thermoelastoplastic deformation of a circular plate due to a partially distributed heat supply," *International Journal of Engineering and Innovative Technology (IJEIT)*, Vol. 3, Issue 4, pp. 396–400, 2013.
- [8] C. M. Jadhav and N. W. Khobragade, "An Inverse Thermoelastic Problem of Thin Finite Rectangular Plate Due to Internal Heat Sources," *International journal of engineering research & technology (IJERT)*, Vol. 2, Issue 6, pp. 1009–1019, 2013, doi: [10.17577/IJERTV2IS60464](https://doi.org/10.17577/IJERTV2IS60464).
- [9] V. Kozlov, V. Mazya and A. Fomin, "The inverse problem of coupled thermo-elasticity," *Inverse Problems*, Vol. 10, No 1, pp. 153–160, 1994, doi: <https://doi.org/10.1088/0266-5611/10/1/012>.

- [10] N. W. Khobragade and K. C. Deshmukh, "An inverse quasi-static thermal deflection problem for a thin clamped circular plate," *Journal of Thermal Stresses*, Vol. 28 (4), pp. 353–361, 2003, doi: <https://doi.org/10.1080/01495730590916605>.
- [11] N. W. Khobragade, P. Hiranwar, H. S. Roy and L. Khalsa, "Thermal deflection of a thick clamped rectangular plate," *Int. J. of Eng. And Innovative Technology*, Vol. 3, Issue 1, pp. 346–348, 2013.
- [12] N. K. Lamba and N.W. Khobragade, "Thermoelastic problem of a thin rectangular plate due to partially distributed heat supply," *IJAMM*, Vol. 8, No. 5, pp. 1–11, 2012.
- [13] R. Ghume and N. W. Khobragade, "Deflection of a thick rectangular plate", *Canadian Journal on Science and Eng. Mathematics Research*, Vol. 3 No. 2, pp. 61–64, 2012.
- [14] R. Roy, S. H. Bagade and N. W. Khobragade, "Thermal stresses of a semi infinite rectangular beam", *Int. J. of Eng. And Innovative Technology*, Vol. 3, Issue 1, pp. 442–445, 2013.
- [15] Shalu D. Barai, M. S. Warbhe and N. W. Khobragade, "Inverse Steady-State Thermoelastic Problems of Semi-Infinite Rectangular Plate," *IJLTEMAS*, Vol. VII, Issue II, pp. 1–6, 2018. [Online]. Available: <https://www.ijltemas.in/DigitalLibrary/Vol.7Issue2/01-06.pdf>.
- [16] Shalu D. Barai, M. S. Warbhe and N. W. Khobragade, "Inverse Transient Thermoelastic problem of Semi-Infinite Rectangular Plate," *IJLTEMAS*, Vol. VII, Issue II, pp. 11–15, 2018. [Online]. Available: <https://www.ijltemas.in/DigitalLibrary/Vol.7Issue2/11-15.pdf>.
- [17] S. S. Singru and N. W. Khobragade, "Thermal Stress Analysis of a Thin Rectangular Plate With Internal Heat Source," *International Journal of Latest Technology in Engineering, Management & Applied Science*, Vol. VI, Issue III, pp. 31–33, 2017. [Online]. Available: <https://www.ijltemas.in/DigitalLibrary/Vol.6Issue3/31-33.pdf>.
- [18] S. S. Singru and N. W. Khobragade, "Thermal Stresses of a Semi-Infinite Rectangular Slab with Internal Heat Generation," *International Journal of Latest Technology in Engineering, Management & Applied Science*, Vol. VI, Issue III, pp. 26–28, 2017. [Online]. Available: <https://www.ijltemas.in/DigitalLibrary/Vol.6Issue3/26-28.pdf>.
- [19] C. S. Sutar and N. W. Khobragade, "An inverse thermoelastic problem of heat conduction with internal heat generation for the rectangular plate," *Canadian Journal of Science & Engineering Mathematics*, Vol. 3, No. 5, pp. 198–201, 2012.
- [20] Y. Tanigawa and Y. Komatsubara, "Thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field," *Journal of Thermal Stresses*, Vol. 20, pp. 517–542, 1997.
- [21] D. S. Steinberg, "Cooling Techniques for Electronic Equipment", Wiley, New York, pp. 105–108, 2000.
- [22] Marks Standard Handbook for Mechanical Engineers, Heat, 8<sup>th</sup> ed., edited by T. Baumeister, McGraw-Hill, New York, pp. 4–69, 2008.

## Обернені задачі та їх використання для проєктування авіаційного навігаційного обладнання

В. О. Повгородній<sup>1,2</sup>

<sup>1</sup> Національний університет "Київський авіаційний інститут", Київ, Україна

<sup>2</sup> Національний технічний університет України "Київський політехнічний інститут імені Ігоря Сікорського", Київ, Україна

**Анотація.** У статті запропоновано метод визначення максимального теплового навантаження за температурним (тепловим) напруженням, вимірним з певною похибкою, шляхом розв'язання оберненої задачі теплопровідності та термопружності. Визначення максимального теплового навантаження таким же чином, як і регулювання зовнішніх і внутрішніх температурних і потужних навантажень, при яких досягаються температурні напруження або зміщення в елементах конструкції в межах допустимих значень, має важливе теоретичне значення і велику практичну цінність, зокрема для завдань неруйнівного контролю. Доцільним способом знаходження цих величин як функцій часу і геометричних координат є розв'язання обернених задач теплопровідності і термопружності, тобто визначення температурного поля на основі поля температурних напружень. Для отримання стійкого розв'язку оберненої задачі термопружності використовується метод А. Н. Тихонова з ефективним пошуком параметра регуляризації. Функціонал А. Н. Тихонова відображає відхилення температурного напруження, отриманого в результаті спостереження, від того, що розраховано на основі наближеного рішення прямої задачі термопружності методами скінченних і граничних елементів. У цьому функціоналі стабілізуючий функціонал з параметром регуляризації використовується як член до квадрата вказаного відхилення. Пошук параметра регуляризації здійснюється за допомогою алгоритму, аналогічного алгоритму пошуку кореня нелінійного рівняння. Використання в методі функцій впливу дозволяє представляти температуру і температурну напругу в залежності від одного і того ж вектора, що значно полегшує реалізацію ітераційного процесу. Запропонований метод дозволяє, не доводячи досліджуваній об'єкт до руйнування, визначити навантаження, при якому він буде зруйнований, тобто вирішити задачу неруйнівного контролю. Ефективність цього методу полягає в тому, що його застосування зменшує витрати на складні експериментальні дослідження об'єктів і усуває необхідність створення аналітичних методів, що супроводжують ці дослідження.

**Ключові слова:** обернена задача, термічний стрес, функції впливу, сплайн, ідентифікація, регуляризація.