

Digital Twin development of a Turning Machine for Designing a Stability Diagram

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Abstract. The turning machining system as an object of study and the dynamic phenomena that constitute the problem that leads to the occurrence of vibrations are presented. The definition of a vibration-free cutting mode is proposed according to the stability diagram simultaneously for two components of the cutting mode – depth and feed. The mathematical model reflects the machining system as a three-mass one in the form of oscillatory links interconnected by negative feedbacks for elastic displacements and positive feedbacks through the delay function. A mathematical model has been developed in the form of a digital twin, due to its representation in state variables, which allows for modeling by numerical methods. Such a model is the basis of the created software, which allows predicting the behavior of the machining system in time and frequency spaces depending on the initial parameters of the machining system and the cutting mode. To ensure adequacy, the model uses dynamic parameters obtained by experimental modal analysis methods. The created software allows you to design stability diagrams of the machining system in the coordinates “cutting depth – speed” and “feed – speed”. The design is carried out automatically according to the stability criterion, which is based on the analysis of the location of the frequency response hodograph on the complex plane. The adequacy of the developed mathematical model and the created procedures for automatic design of the stability diagram is experimentally confirmed by comparing theoretical results and the roughness of the machined surface. The developed digital model and software allow you to choose a vibration-free cutting mode, which guarantees the required quality of machining at maximum productivity.

Keywords: turning machining system, digital twin, stability diagram, frequency stability criterion.

1. Introduction

Turning operations are the most common in various branches of mechanical engineering in the manufacture of machine parts. Such operations are mainly performed on numerically controlled (CNC) machines, which ensures their versatility, high productivity, repeatability and the ability to achieve submicron accuracy [1]. Since these operations involve the formation of a part by removing the allowance material during the cutting process, which is multi-pass, they are always accompanied by the occurrence of vibrations, which is the main obstacle to achieving the desired productivity. Moreover, the chatter that arise are not caused by external factors of changing cutting conditions, but are regenerative, the nature of which lies in the

internal processes occurring in the machining system during cutting. Therefore, the occurrence of chatter in the machining process is an urgent scientific and technical problem that prevents increasing productivity while ensuring a given machining quality.

Many methods of predicting and eliminating chatter are proposed to ensure the required roughness of the machined surface, high productivity and tool stability [2]. Such methods are divided into passive (PCC – Passive Chatter Control), active (ACC – Active Chatter Control) and methods aimed at selecting a chatter-free cutting mode according to the so-called stability lobe diagram (SLD). Passive or active methods always require some modernization of the equipment, while the use of SLD does not require modernization and is easily implemented on CNC machines. However, the effectiveness of assigning a cutting mode according to SLD strongly depends on the adequacy of the adopted mathematical model of the machining system, the design procedure and the convenience of practical use. The adequacy of the dynamic model is ensured

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either by modal experimental studies or by using an approach based on the representation of the dynamic machining system's receptance [3].

Therefore, the design procedure and the convenience of practical use should be based on a mathematical model in the form of a digital twin, which is embodied in special software adapted for practical use. Therefore, to solve such urgent problems, research is needed: first, to develop a new mathematical model, second, to create its digital twin, third, to implement them in practical software.

2. Analysis of literary sources and statement of the problem

The behavior of machining systems, formalized in the stability diagram, has long been noticed by researchers and has been confirmed by experimental methods. Thus, in the work [4] the results of constructing a stability diagram in the coordinates "cutting depth – cutting speed" for the final milling process are presented. It is possible to conclude about certain generalizing regularities inherent in any cutting process [5], which requires the disclosure of the mechanism of oscillations and the definition of the stability criterion.

The most widely used method of constructing a stability diagram is the method improved by Professor Altintas, who brought it to a certain algorithm [6]. This method, although it is based on the representation of the machining system as closed through negative and positive feedback, is based on the solution of the characteristic equation of the differential equation of motion of the system.

However, the characteristic equation for such systems is by definition transcendental and has no roots, so it is solved by an approximate method, which makes it imperfect. In addition, the proposed algorithm consists of several steps, among which there are non-formalized ones that are difficult to automate for creating a software product for designing a diagram on a computer.

Despite these shortcomings, this method has become widespread. For example, a software product based on this approach is known for modeling the interaction of the tool and the workpiece as a system with one degree of freedom with a given resonant frequency and damping coefficient [7]. The program is a demonstration project of Wolfram with open source code. The results are recommended to be used for setting the cutting mode during turning, but with significant limitations caused by the simplified mathematical model and algorithm that are used as the basis, which creates obstacles to the adequacy of the proposed solutions.

Also, using the stability diagram, it is proposed to control turning when facing parts on a CNC machine tool [8]. An analytical approach to modeling is presented through a block diagram of the system in Simulink. However, the system proposed for modeling is single-mass with one degree of freedom and is used only to compare machining with a constant cutting speed function when facing. It is argued that the use of facing with a constant cutting speed

can stabilize the cutting force and reduce vibrations. However, the proposed solution has limited application and cannot be used when turning a wide class of surfaces of parts in the longitudinal direction.

Since the design of the stability diagram is based on the use of a dynamic model of the technological machining system, many studies have been devoted to determining such characteristics. Moreover, taking into account the complexity of conducting modal analysis, it is proposed to use the analysis of the so-called Receptance Coupling Substructure Analysis (RCSA) [9]. In this approach, experimental data are replaced by a theoretical calculation of the dynamic behavior of the instrument with a fixed limit. Moreover, the number of modes taken into account does not exceed one or two. It is also proposed to extend this method to more complex systems. However, to approximate the results to real controlled systems, the experimental data obtained during the study are more reliable. Therefore, it is advisable to conduct research on the determination of stability diagrams, which will be based on the experiment.

The basics of the dynamics of machining systems from the point of view of stability to vibrations in both the frequency and time domains are presented in [10]. Unified dynamic models of mode coupling and regenerative vibrations are presented. The article concludes with modern problems of stability to vibrations in machining, which remain the main obstacle to increasing the productivity and quality of manufactured parts. The article is of a review nature and can be used for general recommendations on avoiding cutting modes that can provoke the occurrence of chatter in the machining system. Therefore, there is a need to develop research on the dynamics of cutting processes that should have practical significance.

To assess the level of chatter, it is recommended to use the roughness parameters of the machined surface [11]. The effect of damping on the stability of the cutting process during turning was investigated using SLD, the construction of which was performed according to the method [12], and experimental modal analysis was used to determine the maximum and minimum frequencies of natural oscillations and damping coefficients. A conclusion is made about the correlation of roughness parameters and vibration level, which is advisable to use in production conditions. However, no specific data are provided on the relationship between the distance of the cutting mode from the limit of stability and the random component of surface roughness, which complicates practical use.

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ver, no specific data are provided on the relationship between the distance of the cutting mode from the limit of stability and the random component of surface roughness, which complicates practical use.

Regenerative vibrations are one of the factors that affect the reduction of cutting efficiency, especially when machining low-rigidity machine parts [14]. The authors rightly consider machining after the trace to be the main cause of such oscillations. The occurrence of regenerative oscillations can be limited by the correct choice of technological parameters, namely, the cutting mode, which is recommended to be determined by the stability diagram. To construct the stability diagram, the well-known [6] formula for the limiting cutting depth is used. This approach limits the dynamics of the machining system to a single-mass model, and the determination of its frequency characteristics is performed by a calculation method using the inverse receptance method, which, of course, reduces the adequacy of the design. Also, when determining the stability diagram, the force characteristics of the cutting process are not taken into account, which also reduces the efficiency of control. However, the paper presents experimental experience of implementing the proposed control on a CNC lathe using a single-board computer embedded in the CNC system of the machine, which is positive for further implementation of theoretical research.

A comprehensive review of vibration suppression strategies with an emphasis on detection, modeling and classification of suppression methods is considered in the article [15]. Both active and passive methods are investigated, including digital signal processing, and the use of stability diagrams for vibration prediction. Experimental and modeling-based approaches to assessing the dynamic behavior of toolholders are also discussed. It is proposed to evaluate the effectiveness of these methods according to four criteria (suppression ability, ease of application, usability and maturity level) on a five-point scale with a special emphasis on boring and turning studies from the 1980s to 2024. When considering the methods of designing a stability diagram, the authors essentially propose one method based on approximate solutions [6], which clearly narrows the possibilities of its practical use. Therefore, the problem of developing a method for designing a stability diagram that is fully adapted to computer design remains relevant.

In [16], a method for detecting the characteristics of vibration oscillations of a machining system at the beginning of machining on a machine tool using signals received using a microphone is proposed. Based on the information obtained, a stability petal diagram was constructed, according to which the cutting mode is searched for in the stability zone. However, despite the presence of signs of frequency characteristics in the sound signal, this is still not enough to assess the stability conditions of the entire machining system.

The characteristics of any dynamic system under operating conditions differ from the characteristics obtained in a static state. Therefore, it is desirable to obtain characteristics, in particular frequency characteristics, under operating conditions. It is proposed to use operational

modal analysis [17], with the elimination of its main drawback, which does not allow determining modal parameters normally distributed by design. A new method is proposed using additional experimental modal analysis to compensate for the missing information. The method adapts to changes in dynamic characteristics under operating conditions. Although it can be considered that for progressing to determining the stability diagram directly in the cutting process with a simplified representation of the dynamics of the machining system in the form of a single-mass system, the operational method is quite sufficient.

A study of the method for calculating static and dynamic forces in cylindrical turning using the analytical method is presented in [18]. Thanks to the analysis of the parameters of the tool vibration characteristics using the fast Fourier transform and the cross-power spectrum, it is shown that the peak of the cutting tool vibration increases with an increase in the cutting volume. A stability analysis of a two-degree-of-freedom system was performed to assess the influence of various parameters on the dynamic cutting force. It is argued that the developed theoretical and experimental studies provide a theoretical and experimental basis for engineering applications. The problem of the adequacy of the proposed solutions remains when a two-coordinate cutting process model is used, although cutting removes a volume of material having a 3D dimension. It is recommended to evaluate the cutting force coefficients in the frequency domain with the development of digital models of all stages of the part production [19]. The intersection of the tool and the workpiece along the tool path is evaluated in discrete steps. This approach is then used to calculate the chip area, cutting force, torque, power and energy consumed by the machine tool, and to detect the occurrence of vibration. The dynamics of the machining system are included in the digital model to estimate the true tangential feed and machining cycle time. Thus, the use of chip area instead of volume as one of the criteria narrows the effectiveness of the proposed approach, which is the basis for expanding the range of research.

The presented review of research gives grounds to argue that it is advisable to conduct research on the development of new models of turning processing systems based on their structural representation, with the subsequent creation of an algorithm for automatic design of stability diagrams.

3. Purpose and objectives of the study

The purpose of the study is to create a mathematical model in the form of a digital twin, suitable for use in computer modeling and design of diagrams of constancy of cutting depth and feed versus spindle speed, which will provide the possibility of assigning a vibration-free cutting mode with maximum productivity.

To achieve the goal, the following tasks were set:

- to develop a model of geometric and force interaction of the tool and workpiece taking into account the dynamics of the lathe machining system;

- create a digital twin and software for digital modeling of the behavior of the machining system in time and frequency space;
- developing a procedure for automatically designing a diagram of constancy according to the coordinates of cutting depth – speed and feed – speed;
- conduct experimental testing of theoretical results.

4. Materials and methods of research

The object of research is the process of turning machining taking into account the elastic machining system with negative feedbacks for elastic displacements in the corresponding coordinates and machining in the wake. In this case, the following assumptions were used: the machining system is represented with two inputs – for the cutting depth and feed, and machining in the wake is implemented by positive feedbacks through the functions of the delay argument. The mathematical model of the machining system is implemented in its system representation in the form of a structure of separate blocks, which allows obtaining a model in differential equations in state variables adapted to numerical modeling and creating a digital twin.

When analyzing the stability, the approach of simultaneous consideration of diagrams in two inputs was applied, which significantly increased the adequacy of the decisions for a chatter-free cutting mode. The analysis of the stability of the obtained solutions is based on the results of modeling the behavior of the machining system in time and frequency spaces.

To develop the software, an object-oriented programming language (Delphi) with standard integration and frequency analysis procedures was used. The procedures are adapted to the features of using the delay argument functions in accordance with the machining system model. The adequacy of the model is ensured by using the results of experimental modal analysis and experimental measurements of the rigidity of the machining system in the direction of the coordinate axes directly on the machine tool.

Experimental verification of the theoretically obtained results was performed on a lathe with a modernized tool holder. The modernization consisted in using the MCS 10 dynamometric sensor in the tool holder, which made it possible to identify the model of the dependence of the cutting force on the cutting mode.

The effectiveness of the developed SLD in ensuring a vibration-free mode was assessed indirectly by the roughness of the machined surface in different locations of the part in terms of the Z coordinate. A change in coordinate causes a predictable change in stiffness, which allows you to monitor the correctness of the response of the created digital twin to changes in cutting conditions.

5. Creation of a digital twin and its use in designing a diagram of constancy by depth and feed

5.1. Model of geometric and force interaction of the tool and workpiece

The model of the machining system of a lathe must adequately reflect its main properties, which are the closedness of the cutting process through negative and positive feedback. Moreover, negative feedback occurs due to elastic displacement due to the action of the cutting force, and positive feedback through the delay function reflects the machining in the wake. The dynamics of the machining system must reflect the oscillatory nature of its behavior in the direction of each coordinate axis, i.e. in 3D space. Therefore, in the mathematical model, at least, the presence of a single-mass system with three degrees of freedom is significant. All feedbacks operating in the machining system are natural and their consideration guarantees the adequacy of the model [20].

The main disturbance of the machining system is the cutting force, which arises as a result of the interaction of the tool and the workpiece, therefore it is advisable to present the calculation scheme in 3D space in two projections (Fig. 1).

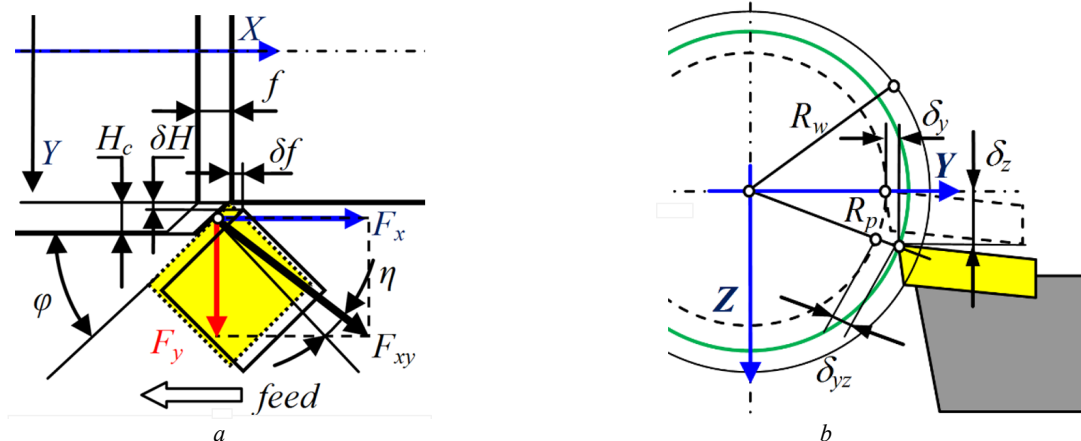


Fig. 1. Calculation scheme: *a* – components of cutting force; *b* – elastic displacements

where $b_0 = \frac{k_{\phi y}}{k_y \omega_z^2} + \frac{k_{\delta z}}{k_z \omega_y^2}$, $b_1 = \frac{k_{\phi y} 2\xi_y}{k_y \omega_z} + \frac{k_{\delta z} 2\xi_z}{k_z \omega_y}$,
 $b_2 = \frac{k_{\delta z}}{k_z} + \frac{k_{\phi y}}{k_y}$, $a_0 = \frac{1}{\omega_y^2 \omega_z^2}$, $a_1 = \frac{2\xi_y}{\omega_y \omega_z^2} + \frac{2\xi_z}{\omega_y^2 \omega_z}$,
 $a_2 = \frac{2\xi_y 2\xi_z}{\omega_y \omega_z} + \frac{1}{\omega_y^2}$, $a_3 = \frac{2\xi_y}{\omega_z} + \frac{2\xi_z}{\omega_y}$, k_y, k_z – stiffness of the elastic machining system along the Y and Z axes, respectively, ω_y, ω_z are the natural oscillation frequencies, ξ_y, ξ_z are the oscillation damping coefficients along the corresponding axes.

Based on the unjustified increase in the order of the model and using the possibility of taking into account the influence of the shift along the Z axis on the change in the diameter of the part in the process of digital modeling, it is advisable (permissible) to simplify the transfer function $W_3(s)$:

$$W_3(s) = k_{\phi y} W_y(s) = \frac{k_{\phi y} / k_y}{\frac{s^2}{\omega_y^2} + 2\xi_y \frac{s}{\omega_y} + 1}. \quad (9)$$

The effect of the shift δ_z along the Z axis can be taken into account in digital modeling according to the following algorithm. At each step of calculating the elastic shift δ_y , using formulas (3), (4), it is possible to predict the component F_z of the cutting force, and $\delta_z = F_z/k_z$. Then, further modeling is performed taking into account the obtained result. It is expected that such a technique will be adequate for a sufficiently small modeling step.

Next, according to the proposed concept of designing a stability diagram, which shows the region of a stable cutting mode, it is assumed to determine the transfer functions of the processing system at the input of the cutting depth H and the feed f .

To determine the transfer function at the input H in the structural diagram of Fig. 2, according to the classical definition of the transfer function, it is necessary to take $f=0$. Then, from the structural diagram of Fig. 2, it is possible to determine the desired transfer function $W_H(s)$, using the rules for transforming structural diagrams:

$$W_H(s) = \frac{\delta_{y1}(s)}{H_1(s)} = \frac{k_H W_1(s) W_3(s)}{1 + k_H W_1(s) W_3(s)}. \quad (10)$$

To determine the transfer function $W_1(s)$, we should return to the original structural diagram in Fig. 2. From the transformed diagram:

$$W_1(s) = \frac{1}{1 + k_{\phi x} W_x(s) k_f}, \quad (11)$$

where $k_{\phi x} = k_{xy} \sin(\varphi + \eta)$ is the coefficient of transformation of the cutting force into the component acting along the X axis.

After substituting the transfer function of the second-order oscillatory link, which represents the dynamics of the machining system in the direction of the X axis, and the necessary algebraic transformations, it can obtain:

$$W_1(s) = \frac{\frac{s^2}{\omega_x^2} + 2\xi_x \frac{s}{\omega_x} + 1}{\frac{s^2}{\omega_x^2} + 2\xi_x \frac{s}{\omega_x} + 1 + \frac{k_f k_{\phi x}}{k_x}}, \quad (12)$$

where k_x is the stiffness, ω_x is the natural frequency of oscillations, ξ is the damping coefficient of oscillations.

Now the transfer function (10) taking into account (9) and (12) after transformations can be represented in standard form:

$$W_H(s) = \frac{Th_{00}s^2 + Th_{01}s + kh}{Th_{10}s^4 + Th_{11}s^3 + Th_{12}s^2 + Th_{13}s + 1}, \quad (13)$$

where $Th_{00} = \frac{k_{\phi y} k_H}{k_y \omega_x^2 Z_h}$, $Th_{01} = \frac{2\xi_x k_H k_{\phi y}}{k_y \omega_x Z_h}$, $kh_0 = \frac{k_H k_{\phi y}}{k_y Z_h}$,

$$Th_{10} = \frac{1}{\omega_x^2 \omega_y^2 Z_h}, \quad Th_{11} = \left(\frac{2\xi_x}{\omega_x \omega_y^2} + \frac{2\xi_y}{\omega_x^2 \omega_y} \right) / Z_h,$$

$$Th_{12} = \left(\frac{1}{\omega_y^2} + \frac{k_f k_{\phi x}}{k_x \omega_y^2} + 4\xi^2 \frac{1}{\omega_x \omega_y} + \frac{1}{\omega_x^2} + \frac{k_H k_{\phi y}}{\omega_x^2 k_y} \right) / Z_h,$$

$$Th_{13} = \left(\frac{2\xi}{\omega_y} + \frac{k_f k_{\phi y} 2\xi_x}{k_x \omega_y} + \frac{2\xi_y}{\omega_x} + \frac{k_H 2\xi_x k_{\phi y}}{k_y \omega_x} \right) / Z_h,$$

$$Z_h = \frac{k_x k_y + k_f k_{\phi x} k_y + k_H k_{\phi y} k_x}{k_x k_y}.$$

To determine the transfer function at the input f in the structural diagram of Fig. 2, similarly to the previous method, it is necessary to take $H=0$. Then, using the rules for transforming structural diagrams, it can obtain the desired transfer function of the system at the input of the feed:

$$W_f(s) = \frac{k_f W_2(s) k_{\phi x} W_x(s)}{1 + k_f W_2(s) k_{\phi x} W_x(s)}, \quad (14)$$

where the transfer function $W_2(s)$ is determined from the structural diagram of Fig. 2 under the condition of simplifying the influence of the shift δ_z along the Z axis on the change in the radius of the part due to dynamic processes:

$$W_2(s) = \frac{1}{1 + k_{\phi y} W_y(s) k_H}, \quad (15)$$

where $k_{\phi y} = k_{xy} \cos(\varphi + \eta)$ is the coefficient of transformation of the cutting force into the component acting along the Y axis.

After substituting the transfer function of the second-order oscillatory link, which represents the dynamics of the machining system in the direction of the Y axis and the necessary algebraic transformations, it can obtain:

$$W_2(s) = \frac{\frac{s^2}{\omega_y^2} + 2\xi_y \frac{s}{\omega_y} + 1}{\frac{s^2}{\omega_y^2} + 2\xi_y \frac{s}{\omega_y} + 1 + \frac{k_H k_{\phi y}}{k_y}}, \quad (16)$$

where k_y is the stiffness in the direction of the Y axis, ω_y is the natural frequency of oscillations, ξ_y is the oscillation damping coefficient.

Now the transfer function (14) taking into account (16), after transformations can be represented in the standard form:

$$W_f(s) = \frac{Tf_{00}s^2 + Tf_{01}s + kf}{Tf_{10}s^4 + Tf_{11}s^3 + Tf_{12}s^2 + Tf_{13}s + 1}, \quad (17)$$

$$\text{where } Tf_{00} = \frac{k_{\varphi x} k_f}{k_x \omega_y^2 Z_f}, \quad Tf_{01} = \frac{2\xi_y k_f k_{\varphi x}}{k_x \omega_y Z_f}, \quad kf_0 = \frac{k_f k_{\varphi x}}{k_x Z_f},$$

$$Tf_{10} = \frac{1}{\omega_x^2 \omega_y^2 Z_f}, \quad Tf_{11} = \left(\frac{2\xi_x}{\omega_y \omega_x^2} + \frac{2\xi_y}{\omega_y^2 \omega_x} \right) / Z_f,$$

$$Tf_{12} = \left(\frac{1}{\omega_x^2} + \frac{k_H k_{\varphi y}}{k_y \omega_x^2} + 4\xi_y^2 \frac{1}{\omega_x \omega_y} + \frac{1}{\omega_y^2} + \frac{k_f k_{\varphi x}}{\omega_y^2 k_x} \right) / Z_f,$$

$$Tf_{13} = \left(\frac{2\xi_x}{\omega_x} + \frac{k_H k_{\varphi y} 2\xi_x}{k_y \omega_x} + \frac{2\xi_y}{\omega_y} + \frac{k_f 2\xi_x k_{\varphi x}}{k_x \omega_y} \right) / Z_f,$$

$$Z_f = \frac{k_x k_y + k_H k_{\varphi y} k_x + k_f k_{\varphi x} k_y}{k_x k_y}.$$

It is this form of representation of the mathematical model that allows you to obtain a digital twin of the processing system, which allows you to perform computer modeling in both time and frequency spaces.

Therefore, for the practical application of the mathematical model of the processing system, which reflects real processes and can be used to design a stability diagram, it is necessary to identify the corresponding dynamic characteristics. Since the model presents three oscillatory links, it is necessary to determine the frequencies of their natural oscillations, the coefficients of oscillation damping and stiffness along all three coordinate axes. In addition, it

is necessary to determine the coefficients of the force dependences of the cutting process and the geometric parameters of the cutter and the workpiece. All such parameters and characteristics can be identified using the method presented in [23].

5. 2. Simulation the machining process

To model the cutting process in a turning machining system using an object-oriented programming language, an application program was created in which the mathematical model according to Fig. 2 is implemented. The modeling is performed using the initial data of the machining system, which are presented in the interface windows, which allows their prompt change and study of the influence of each parameter of the machining system on the process (Fig. 3).

The interface of the developed modeling program also presents modeling control buttons, by which you can choose to demonstrate the process in the time or frequency domain. The determination of all other relevant data of the machining system being modeled is performed using the experimental modal analysis method [23], which guarantees the adequacy of the obtained results.

To simulate processes in time, the software uses the numerical integration procedure sequentially for each dynamic model for the inputs of the depth (Y coordinate) and feed (X coordinate) of cutting. Numerical integration is performed over time at each step in accordance with the 4th order Runge-Kutta algorithm.

Other parameters of the machining system that determine the process of cutting the workpiece correspond to the material Steel 30, specific cutting force 1450 N/mm², length when fixed in the chuck and back center 250 mm, diameter 30 mm. A virtual oscilloscope based on the Delphi

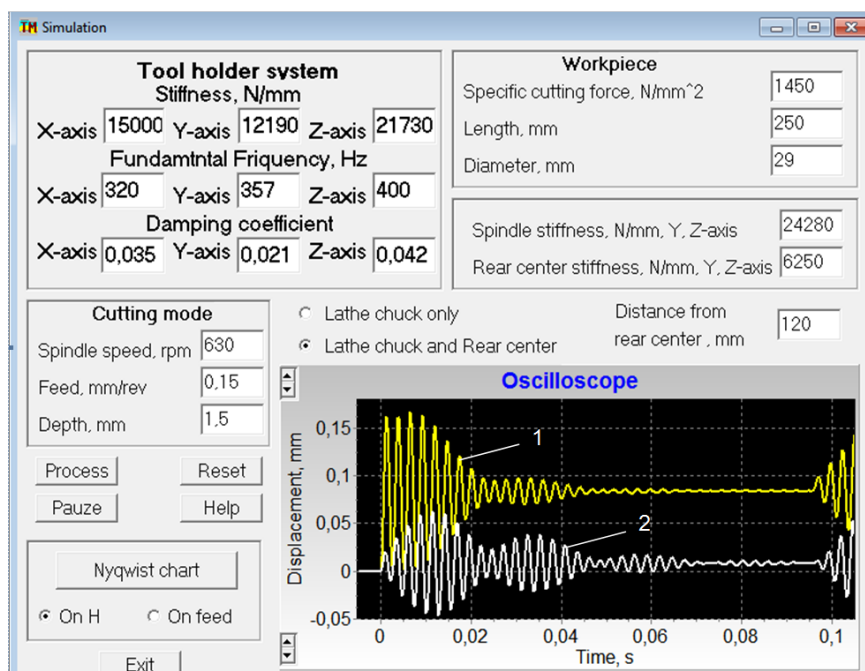


Fig. 3. Interface with the output data of the machining system

Chart component was built into the interface of the created software, where the response of the machining system over time is presented in the form of oscillograms of elastic displacements in the Y and X coordinates.

Fig. 3 shows the oscillograms of elastic displacements of the machining system when simulating the following cutting mode: spindle speed 630 rpm cutting depth 1.5 mm, feed 0.15 mm/rev. Taking into account the dependence of dynamic processes in the machining system on the stiffness of the workpiece, which varies in length, the simulation was performed for positions at a distance of 120 mm from the back center. The oscillations of the machining system in the Y (line 1) and X (line 2) coordinates at the beginning of the process are presented, which gradually decay. You can observe the restoration of oscillations after the end of the first revolution of the workpiece in accordance with the reaction to subsequent machining.

To assess the stability of the processing system, it is advisable to use the criterion proposed for systems with positive feedback through the function of the lagging argument [22]. The criterion is based on the location of the Nyquist diagram on the complex plane: the system is stable if the diagram does not cover a point with coordinates $[+1; 0]$ (Fig. 4). The created computer program allows you to determine such characteristics by two coordinates.

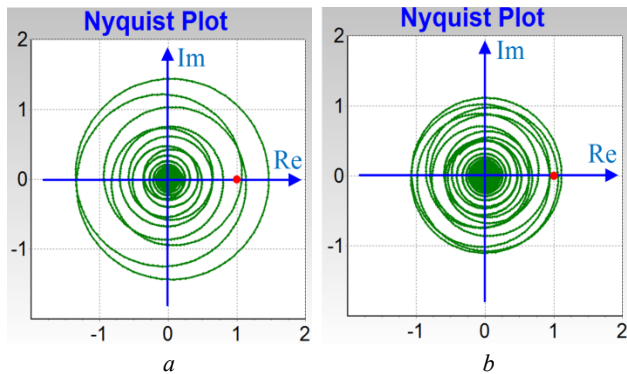


Fig. 4. Nyquist diagrams: a – along the Y coordinate; b – along the X coordinate

Fig. 4 presents the Nyquist diagrams of the machining system along two inputs. Both diagrams cover the point with coordinates $[+1, 0]$ (highlighted in red), which, in accordance with the proposed stability criterion, indicates the instability of the machining system. Therefore, the machining system under such cutting modes is not stable along both directions of elastic shear motion. Continuation of the simulation in time space (Fig. 5) confirms the following conclusion: the amplitudes of both oscillograms increase. Instability along one coordinate also indicates the instability of the entire system.

Reducing the cutting depth to 1 mm with other unchanged modes quite expectedly ensures a stable cutting process. This is evidenced by the location of the Nyquist diagrams on the complex plane (Fig. 6) along two coordinates and the course of elastic displacements of the processing system when simulating the process in time through the same time interval as in the previous experiment (Fig. 6).

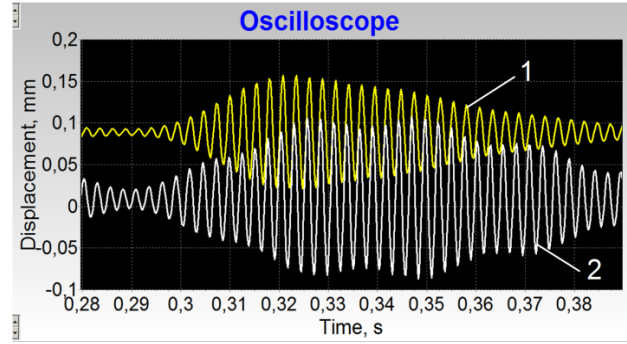


Fig. 5. Oscillograms of the process after 0.3 s

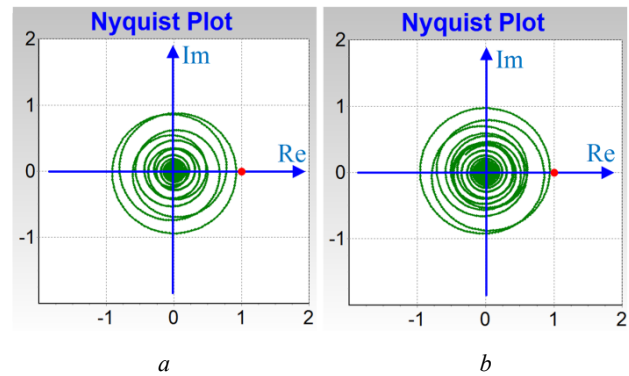


Fig. 6. Nyquist diagrams: a – along the Y coordinate; b – along the X coordinate

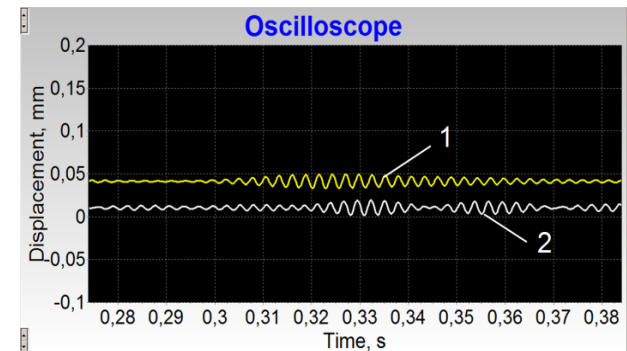


Fig. 7. Oscillograms of the process after 0.3 s

The software created in this way is a tool for studying the prediction of the dynamic behavior of the machining system depending on its initial data. The results obtained quite adequately represent the response of the machining system in qualitative terms. The adequacy of the assessment of the created software in quantitative terms can be confirmed by the results of experimental studies. Thus, the developed digital twin of the machining system can be used as the basis for the second part of the created software, which performs automatic design of stability diagrams.

5. 3. Design of the stability diagram

The created software provides an additional interface for controlling the process of designing the stability

diagram. The design is carried out automatically for the entire specified range of spindle speed according to a new criterion, which is based on the location of the hodograph of the frequency transfer function on the complex plane [22]. The designed diagram can be saved in digital format for each combination of cutting modes, which allows their further representation in the corresponding coordinates.

Fig. 8 shows the stability diagrams in the coordinates “spindle speed – feed” at different cutting depths. The area above the graph of each diagram corresponds to the unstable mode of the machining system and, accordingly, below the graph – the stable mode.

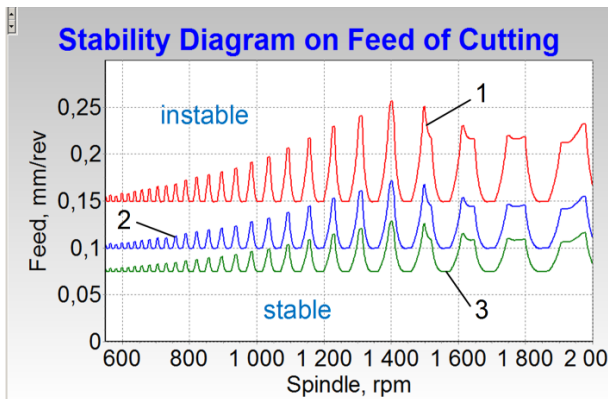


Fig. 8. Stability diagrams by feed: 1 – cutting depth 1 mm, 2 – cutting depth 1.5 mm, 3 – cutting depth 2 mm

Since the graphs of the diagrams are constructed according to a special algorithm based on the new criterion of constancy, and not on the traditional approach that uses the construction of “petals” [6], their name has been changed. Despite the proposed change in the name (instead of “petal constancy diagram” simply “constancy diagram”), the periodic nature of the diagram is preserved, which indicates an adequate representation of the physics of dynamic processes in both cases. Thus, the proposed approach to determining the constancy diagram retains the repeated confirmation of the periodicity of dynamic processes during cutting, although, unlike the previously used one, it allows for the automation of design.

The representation of the mathematical model in the proposed form, which is based on the concept of a transfer function, allows not only determine the transient, but also the frequency characteristics of the machining system simultaneously in two coordinates. Using this property, it is also possible to design stability diagrams in the *Y* coordinate, which corresponds to the cutting depth. Fig. 9 shows the stability diagrams constructed in automatic mode by the created software in the cutting depth at different feeds.

Increasing the feed (Fig. 9), as well as the cutting depth (Fig. 8) narrows the area of stable cutting of the machining system. The developed software allows you to determine stable cutting modes during turning for different initial characteristics of machining systems and different ranges of machining modes, which makes it an effective tool for a programmer.

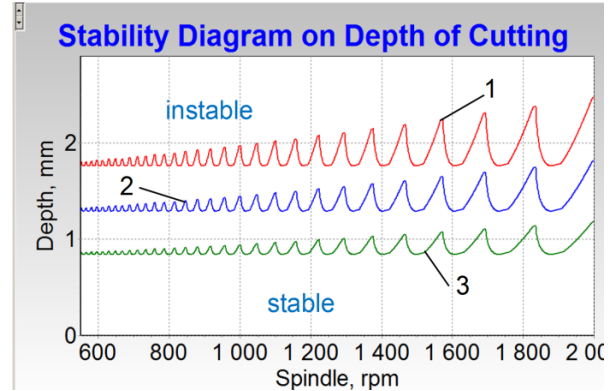


Fig. 9. Depth stability diagrams: 1 – feed 0.1 mm/rev; 2 – feed 0.15 mm/rev, 3 – feed 0.25 mm/rev

5. 4. Experimental verification of the adequacy of the developed software

The adequacy check was performed during the comparison of the predicted results with the experimental ones when machining workpiece on a machine with the parameters determined during modeling. It should be noted that the theoretically designed diagram represents the boundary of stable operation of the machining system. However, since the mathematical model used for the design is always some approximation of real processes, the stability assessment should be performed within a certain band, the width of which is determined by the adequacy of the model. Thus, to assess the degree of stability of the cutting process, it is advisable to use an indirect indicator of vibration intensity based on the roughness of the machined surface. As already noted [11], [12], this approach is well proven and can be used in this work.

Taking into account the dependence of the location of the stability diagram on the dynamic parameters of the machining system, it was decided to assess the influence of rigidity on the vibration level using the developed software. For this purpose, a 250 mm long workpiece was machined, fixed in the chuck and tailstock, at a spindle speed of 630 rpm, a feed of 0.15 mm/rev, and a cutting depth of 1.5 mm, and the surface roughness was measured in four places, in the longitudinal direction and in the circular direction, as shown in Fig. 10. The results are presented in Table 1. This ensures the change in stiffness determined in the mathematical model and its response to the constancy of the cutting process in accordance with the defined and designed constancy diagrams.

Table 1. Measurement results

Distance from rear center	On the axis		On the circular		Depth change δH , mm
	<i>Ra</i> , μm	<i>Rz</i> , μm	<i>Ra</i> , μm	<i>Rz</i> , μm	
20 mm	2,22	15,9	1,17	8,15	0,54
70 mm	2,97	15,5	1,16	7,35	0,45
120 mm	2,42	11,9	1,01	6,29	0,38
170 mm	2,16	11,5	0,89	6,0	0,24

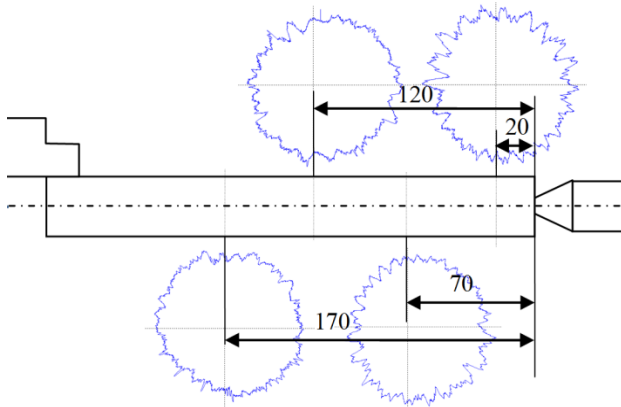


Fig. 10. Profile roughness circular diagrams

Therefore, the assessment of the qualitative adequacy of the developed software product was based on the correlation of the roughness measured in the longitudinal and transverse directions with the results of the location of the stability diagrams along the Y coordinate from the given cutting depth. Fig. 11 shows a fragment of the stability diagrams along the cutting depth, enlarged from Fig. 9 in the area of the given spindle speed of 630 rpm. The stability diagrams are designed for the rigidity of the machining system in the sections of the workpiece: line 1 – at a distance of 20 mm from the back center, line 2 – 70 mm, line 3 – 120 mm and line 4 – 170 mm in accordance with the scheme of Fig. 10.

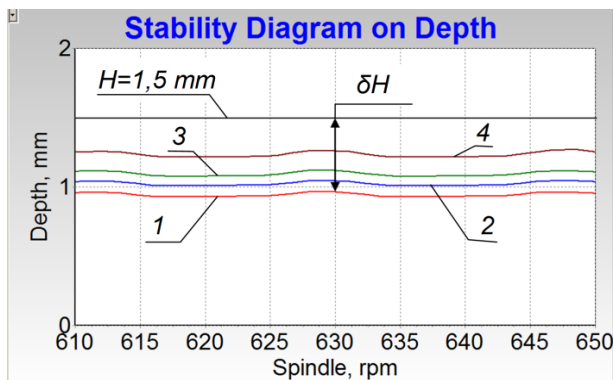


Fig. 11. Fragment of stability diagrams by cutting depth

A qualitative assessment of the influence of changes in hardness on the roughness of the machined surface can be carried out by the distance to each designed diagram and compared with the measured roughness. That is, to ensure the surface roughness that will correspond to the stability limit determined by the designed diagram, it is necessary to reduce the cutting depth by the value δH (see table 1) compared to that set when machining the specified areas. The same result can be obtained from the analysis of the stability diagrams by feed (Fig. 8).

Analysis of the data in table 1 and the results from Fig. 11 confirm the adequate response of the developed model to changes in the dynamic parameters of the machi-

ning system, namely its rigidity. The cutting process is entirely located in an unstable region, therefore the parameter Rz is much larger than the deterministic component, which is determined by the geometric interaction of the cutter and the workpiece.

A qualitative assessment of the impact of changes in hardness on the roughness of the machined surface can be carried out by the distance to each diagram and compared with the measured roughness. That is, to ensure surface roughness that will correspond to the stability limit determined by the stability graph, it is necessary to reduce the cutting depth by the value δH (see table 1) compared to the specified one. Such a quantitative assessment corresponds to the results of experiments on the measured roughness of the surface of the part in specified places.

A greater distance from the specified cutting depth to the corresponding boundary of the diagram is sufficiently correlated with the measured roughness in both the longitudinal and transverse (circular) directions. The further the roughness measurement area is located from the back center, where the rigidity of the machining system increases according to the model, the less the influence of vibrations on the roughness.

6. Discussion of the results of creating a digital twin of a turning machining system

Therefore, a vibration-free cutting mode can be ensured by stability diagrams, which are determined by the mathematical model of the machining system in the form of a digital twin, which reflects the dynamics of a three-mass system in the direction of three coordinates. At the same time, to ensure the adequacy of the results, it is recommended to determine the frequencies of natural oscillations in each direction according to the results of experimental modal analysis, in contrast to the reception methods [9]. It is also necessary to determine the dependence of the cutting force and its components on the cutting mode and conditions, which can be attributed to the shortcomings that complicate the possible on-line algorithm for determining the stability diagram. However, building a mathematical model in the structural diagram, creating a digital twin and applying a new stability criterion in the form of the location of the Nyquist diagram of the entire system on a complex plane, in contrast to the traditional ones [6], gives real chances to solve such a problem.

The developed mathematical model according to the structural scheme of Fig. 2 takes into account the dynamics of the machining system for three masses. In accordance with the scheme of elastic displacements (Fig. 1), the influence of the elastic displacement along the Z axis on the actual cutting depth, which is measured along the Y axis, is taken into account for the first time. The influence is taken into account by the coefficient of the linearized dependence (6), which generally increases the adequacy of the model, since the displacement along the Z coordinate should affect the actual machining diameter.

The main thing that makes the developed structure as close as possible to taking into account the main cause

of regenerative oscillations is the use of positive feedback through the function of the lagging argument along both coordinates X and Y . Such a connection is reflected in the structural scheme (Fig. 2) and is implemented in the digital twin by adding an array of displacement data along the coordinates on the previous rotation of the part to the current values.

Representation of the mathematical model through transfer functions (13) and (17) provides the possibility of modeling in time and frequency space, and the use of representation in the model through state variables makes the model a real digital twin. It is this representation that makes it possible to use standard procedures for calculating the frequency characteristics of the processing system, without which it is impossible to use the stability criterion proposed in [22].

For the practical application of the digital twin when solving the problem of choosing a cutting mode in the process of technological preparation of production, application software was created. With its help, it is possible to perform digital modeling and predict the quality of the processing process depending on the course of the process in time, and to estimate the stability margin of the processing system according to the new criterion presented for the first time in [22].

To solve the technological problem of assigning a cutting mode, the software provides procedures for designing diagrams of the stability of the machining system, and in two coordinates simultaneously, corresponding to the depth and feed of the cut. Such design is performed automatically and does not require entering additional data or performing additional operations, unlike other existing algorithms [6]. The diagrams are presented in a form convenient for practical use by a technologist-programmer (Fig. 8 and Fig. 9) when assigning a cutting mode in control programs for CNC machines. To assess the correctness of the mathematical model, it is proposed to use the indirect effect of vibrations in the machining system on the roughness of the surface of the part. To confirm the adequacy, the effect of the dynamic parameters of the system on the model and comparison with the consequences that affect the roughness were used. Taking into account the change in the rigidity of the entire machining system when the cutter moves along the length of the part, diagrams of stability were designed for each section, in which the roughness was then measured. The correlation found between the distance in cutting depth from the mode to each stability diagram and the surface roughness indirectly confirms the adequacy of the model.

Thus, the developed digital twin of the turning processing system forms the basis of the created software for designing a control program for a CNC machine tool, which guarantees vibration-free processing at maximum productivity.

The limitations in the practical use of the research results are the need to pre-determine the dynamic parameters of the processing system. Such parameters include the stiffness along the coordinate axes, the frequencies of natural oscillations and the damping coefficients of oscillations. Therefore, it is recommended to conduct preliminary

studies (for example, once for a batch of parts) such as the methodology described in [23]. Data on the coefficients of the cutting force model (1) are also required. For their identification, it is recommended to use reference data [21].

The continuation of research in ensuring vibration-free cutting is seen in the creation of operational means of on-line correction of the initially programmed mode. It is advisable to determine the dynamic parameters of the processing system necessary for designing the stability diagram using operational modal analysis methods and sensor signals of a numerically controlled machine tool.

7. Conclusions

1. The developed model of geometric and force interaction of the tool and the workpiece in three coordinates is the basis of the mathematical model of the machining system, which is presented in the form of a structural diagram using transfer functions and feedback. Moreover, negative feedback acts on elastic displacements, and positive feedback - through the functions of the delaying argument. The delay time is set by the spindle rotation frequency.

2. The developed mathematical model in the form of a digital twin, which uses its representation in state variables, is the basis of the created software. The software tool for digital modeling allows you to predict the behavior of the machining system in time and frequency spaces depending on the initial parameters of the machining system and the cutting mode.

3. Using a new criterion for the stability of machining systems, based on determining the location of the Nyquist diagram hodograph on the complex plane, a procedure for automatically designing stability diagrams in two coordinates was created. Thus, in the created software, it is possible to design stability diagrams of the machining system in the coordinates "cutting depth - speed" and "feed - speed".

4. This property of the created software makes it a useful tool for a technologist-programmer when determining a vibration-free cutting mode.

5. Experimental testing of the developed mathematical model and the created procedures for automatically designing the stability diagram was carried out by comparing theoretical results and the roughness of the machined surface. At the same time, the reaction of the created software to a change in the rigidity of the part along the length and the consequences in the form of a change in surface roughness were evaluated. The results obtained confirmed a certain correlation, which indicates the adequacy of the model.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, including financial, personal, authorship, or any other nature that could affect the research and its results presented in this article.

Editorial Process Transparency Statement

One of the authors serves on the Editorial Board of the journal. In accordance with the journal's conflict of interest policy, the author had no involvement in the peer review, editorial assessment, or final decision regarding the acceptance of this manuscript.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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Розробка цифрового двійника токарного верстата для побудови діаграми сталості

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Анотація. Представлена токарна обробна система як об'єкт дослідження і динамічні явища, що складають проблему, яка приводить до виникнення вібрації. Запропоновано визначення безвібраційного режиму різання за діаграмою сталості одночасно за двома складовими режиму різання – глибиною і подачею. Математична модель відображає обробну систему як трьох масову у вигляді коливальних ланок, що з'єднані між собою негативними зворотними зв'язками за пружними зсувами та позитивними зворотними зв'язками через функцію запізнення. Розроблена математична модель у вигляді цифрового двійника, завдяки її представленню у змінних стану, що дає можливість моделювання чисельними методами. Така модель покладена в основу створеного програмного забезпечення, яке дозволяє прогнозувати поведінку обробної системи у часовому та частотному просторах в залежності від вихідних параметрів обробної системи та режиму різання. Для забезпечення адекватності в моделі використовуються динамічні параметри, отримані методами експериментального модального аналізу. Створене програмне забезпечення дозволяє проєктувати діаграми сталості обробної системи в координатах «глибина різання – швидкість» і «подача – швидкість». Проєктування здійснюється автоматично за критерієм сталості, що базується на аналізі розташування годографа частотної характеристики на комплексній площині. Адекватність розробленої математичної моделі і створених процедур автоматичного проєктування діаграми сталості експериментально підтверджена шляхом порівняння теоретичних результатів і шорсткості обробленої поверхні. Розроблена цифрова модель і програмне забезпечення дозволяють обирати безвібраційний режим різання, що гарантує необхідну якість оброблення при максимальній продуктивності.

Ключові слова: токарна обробна система, цифровий двійник, діаграма стійкості, критерій частотної сталості.