UDC 621.9.06-229.33
DOI: http://dx.doi.org/10.20535/2521-1943.2017.79.92754

Static calculation of the “Spindle unit” elastic system by using transfer matrices method

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Received: 12 February 2017 / Accepted: 11 April 2017

Abstract. *Methodology, that allows to use the transfer matrices method for the static calculation of spindle unit as elastic system, which consists of a few associate subsystems, has been developed. Restrictions of the dynamic compliances method use for static calculations of «spindle unit» system decomposing has been stated. For such calculations it is suggested to conduct the decoupling of the system with the use of mixed method of indefinable beam systems static calculation. The elastically-deformation model of the «Spindle unit» system, that consist of subsystems: tool, spindle, spindle housing elastically mounted on the machine-tool bed, is being presented. The scheme of decoupling of this system for static calculations has been developed. The analytical solution of task of the elastic displacements calculation in the characteristic points of subsystems has been obtained. The algorithm of static calculation of the «Spindle unit» elastic system has been developed. Offered approach allows applying an identical methodological base for the static and dynamic characteristics simulation of the «Spindle unit» elastic system.*

Keywords: *metal-cutting machine tool; spindle unit; elastic-deformation model; decoupling of the elastic system, transfer matrices method*

Introduction

Accuracy of machining on metal-cutting machine tools is determined by mutual position of tool and workpiece during machining and first of all depends on the size of their elastic displacements. Therefore the questions related to the increase of accuracy of elastic displacements determination of tool and workpiece in the machining zone are actual.

Possibilities of direct determination of elastic displacements of tool and workpiece in the machining zone are limited. Foremost, it is related to measuring complication of axis displacement of the rotating element: tool at milling or workpiece at turning. As a rule, such displacements are determined by indirect methods, for example, by results of measuring of supports deformations in spindle unit or displacement of spindle pin, with fixed tool or workpiece [1]. It means for determination of elastic displacement of the rotating element in the cutting zone this approach assume the use of calculation elastic-deformation model of spindle unit. Thus in a model necessarily must be taken into account a presence of the fixed tool (workpiece) in spindle.

For developing of such models, the method of finite elements (FEM) is mostly used. For the spindle, spindle unit housing and a tool (workpiece), fixed in spindle, simulations, beam elements of Euler-Bernoulli, Rayleigh or Timoshenko are used. Spindle bearings, joint of tool (workpiece) with spindle and joint of spindle unit housing with the bed of machine-tool - simulated by elastic connections [1, 2]. Such model allows calculation static and dynamic characteristics of the «Spindle unit» system [2]. The use of FEM allows obtaining the most exact results of simulation, but procedure of model development differs in considerable labour intensiveness and complication.

Among another ways of the elastic system description of spindle unit it should be noted the use of the method of initial parameters in matrix formulation, more known as a transfer matrices method (TMM)[3], [4]. The basis of this method is finding solution of natural bending eigentones of beam element of permanent section with uniformly distributed mass [5, 6]. The most simple calculation model of spindle unit, made with the use of this method, appears as the elastic step beam set on elastic-dissipative supports [3]. This model is used for a calculation of static and dynamic characteristics of spindle unit. Compared to FEM, TMM has certain advantages that consist in the following [3]. Direct characteristics participate in a calculation, such as stiffness, damping, that can be determined experimentally, or set from reference books. With a high number of beam elements in the model, solving of the large system of equations is

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not required. Depending on the degree of detailing of model, TMM can be considered as both approximate and précised methods of calculation. Use of TMM in combination with the method of dynamic compliances [7] allow to carry out the calculation of dynamic characteristics of spindle unit as an elastic system consisting of a few associate subsystems [4]. Such model meet the requirements maintained to the calculation models of spindle units [1], but her use is limited to the calculation of only dynamic characteristics.

Aim

The aim of this work is development of methodology, that allow to use the transfer matrices method for the calculation of static characteristics of spindle unit as elastic system consisting of a few associate subsystems.

Methodology of decoupling of the elastic system «Spindle unit» for the calculation of her static characteristics with the use of method of transitional matrices

Methodology of calculation of dynamic characteristics of the elastic system «Spindle unit» supposes [4] her decoupling and solving of the system of deformations compatibility equations in points the of subsystems disconnection. As a result of solution of this equations system the reactions of throw-away connections are determined. Then, for the characteristic points of subsystems canonical equations of method of forces are made. As a result, solving of these equations (taking into account obtained values of reactions of throw-away connections) displacements are determined in the characteristic points of subsystems. On the values of these displacements, frequency characteristics of the system is being calculated, as well as forms of subsystems vibrations on natural frequencies and elastic lines of subsystems, under external load, applied to the system. For equilibrium equations composing in points of the subsystems disconnection the method of dynamic compliances is used. Included in equilibrium equations the harmonic influence coefficients in points of the system disconnection are determined with the use of harmonic coefficients of influence of subsystems that, in turn, are determined with the use of method of initial parameters in a matrix form (transfer matrices method). The general order of harmonic influence coefficients determination of subsystems is described in [4].

As an example, we will consider the elastic system «Spindle unit» as mounted on the machine-tool bed of internal grinding head of quill-type execution [4], [8]. Structural and calculation charts of this system are presented on a fig. 1. Chart of decoupling of this system with the selection of subsystems of tool (subsystem 1, index $s=1$), spindle (subsystem 2, index $s=2$) and spindle unit housing elastically mounted on the machine-tool bed (subsystem 3, index $s=3$) presented on a fig. 2. This chart of decoupling is used for the calculation of dynamic characteristics of the elastic system «Spindle unit».

The system of deformations compatibility equations in points of subsystems disconnection [4], [8] has the form:

$$\begin{cases} \alpha_{00}^{12}\cdot X_0+\gamma_{00}^{12}\cdot M_0-\alpha_{01}^{(2)}\cdot X_1-\alpha_{02}^{(2)}\cdot X_2-\alpha_{03}^{(2)}\cdot X_3-\alpha_{04}^{(2)}\cdot X_4=-\alpha_{20}^{(1)}\cdot P_0^{(1)} \\ \beta_{00}^{12}\cdot X_0+\phi_{00}^{12}\cdot M_0-\beta_{01}^{(2)}\cdot X_1-\beta_{02}^{(2)}\cdot X_2-\beta_{03}^{(2)}\cdot X_3-\beta_{04}^{(2)}\cdot X_4=-\beta_{20}^{(1)}\cdot P_0^{(1)} \\ -\alpha_{10}^{(2)}\cdot X_0-\gamma_{10}^{(2)}\cdot M_0+\alpha_{11}^{23}\cdot X_1+\alpha_{12}^{23}\cdot X_2+\alpha_{13}^{23}\cdot X_3+\alpha_{14}^{23}\cdot X_4=0 \\ -\alpha_{20}^{(2)}\cdot X_0-\gamma_{20}^{(2)}\cdot M_0+\alpha_{21}^{23}\cdot X_1+\alpha_{22}^{23}\cdot X_2+\alpha_{23}^{23}\cdot X_3+\alpha_{24}^{23}\cdot X_4=0 \\ -\alpha_{30}^{(2)}\cdot X_0-\gamma_{30}^{(2)}\cdot M_0+\alpha_{31}^{23}\cdot X_1+\alpha_{32}^{23}\cdot X_2+\alpha_{33}^{23}\cdot X_3+\alpha_{34}^{23}\cdot X_4=0 \\ -\alpha_{40}^{(2)}\cdot X_0-\gamma_{40}^{(2)}\cdot M_0+\alpha_{41}^{23}\cdot X_1+\alpha_{42}^{23}\cdot X_2+\alpha_{43}^{23}\cdot X_3+\alpha_{44}^{23}\cdot X_4=0 \end{cases}, \qquad (1)$$

where X_i, M_i – amplitudes of harmonic reactions of throw-away connections; α_{ij}^{ss}, β_{ij}^{ss}, γ_{ij}^{ss}, ϕ_{ij}^{ss} – harmonic influence coefficients in points of the system decoupling on subsystems s ($s=1,2,3$); $\alpha_{ij}^{(s)}$, $\beta_{ij}^{(s)}$, $\gamma_{ij}^{(s)}$, $\phi_{ij}^{(s)}$ – harmonic influence coefficients of separate subsystems; $P_0^{(1)}$ - external load applied on a tool.

Determinations of harmonic influence coefficients of subsystems is based on the use of transfer matrices of these subsystems $\mathbf{\Pi}^{(s)}$. For our example, matrices $\mathbf{\Pi}^{(s)}$ will be [4]:

for a tool ($s=1$, $u=2$): $\mathbf{\Pi}^{(1)}=\prod_{i=2}^{0}\mathbf{\Pi}_i^{(1)}=\mathbf{U}_2^{(1)}\cdot\mathbf{U}_1^{(1)}\cdot\mathbf{G}_0^{(1)}$; (2)

for the spindle ($s=2$, $u=6$): $\mathbf{\Pi}^{(2)}=\prod_{i=6}^{0}\mathbf{\Pi}_i^{(2)}=\mathbf{U}_6^{(2)}\cdot\mathbf{G}_5^{(2)}\cdot\mathbf{U}_5^{(2)}\cdot\mathbf{U}_4^{(2)}\cdot\mathbf{U}_3^{(2)}\cdot\mathbf{U}_2^{(2)}\cdot\mathbf{U}_1^{(2)}$; (3)

for the spindle unit housing ($s=3$, $u=7$): $\mathbf{\Pi}^{(3)}=\prod_{i=7}^{0}\mathbf{\Pi}_i^{(3)}=\mathbf{U}_7^{(3)}\cdot\mathbf{R}_6^{(3)}\cdot\mathbf{U}_6^{(3)}\cdot\mathbf{U}_5^{(3)}\cdot\mathbf{U}_4^{(3)}\cdot\mathbf{U}_3^{(3)}\cdot\mathbf{U}_2^{(3)}\cdot\mathbf{R}_1^{(3)}\cdot\mathbf{U}_1^{(3)}$. (4)

where for every subsystem: $\mathbf{G}_i$ – mass-inertia matrix of the concentrated load; $\mathbf{R}_i$ – matrix of elastic-dissipative linear and angular support; $\mathbf{U}_i$ – matrix of the beam section with distributed mass; i – number of the subsystem section; u – general number of subsystem sections.

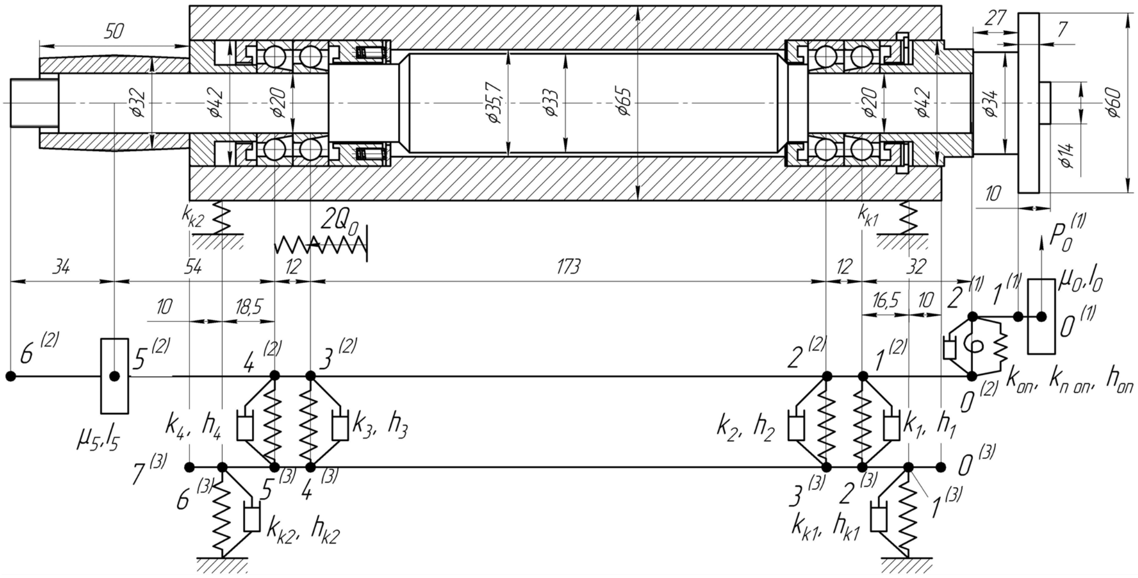

Fig. 1. Structural and calculation charts of the «spindle unit» elastic system

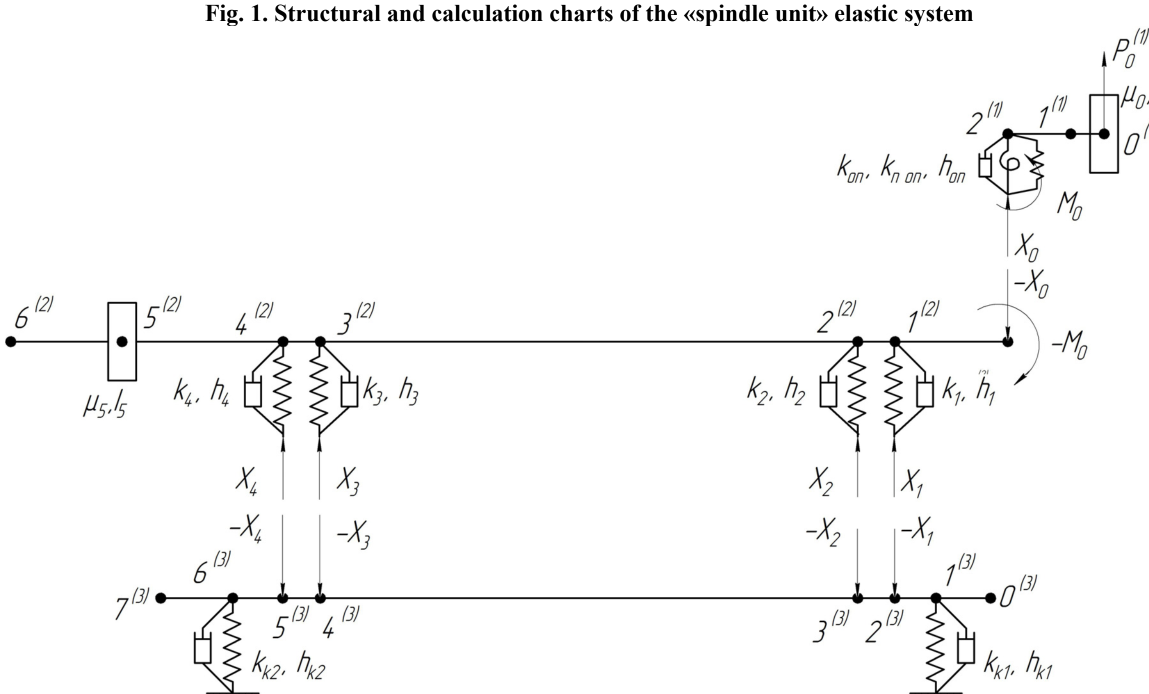

Fig. 2. Chart of decoupling of the elastic system «Spindle unit» for dynamic calculations: subsystem of tool (subsystem 1, index $s=1$); subsystem of spindle (subsystem 2, index $s=2$); subsystem of the spindle housing (subsystem 3, index $s=3$)

The static influence coefficients of subsystems are determined in the same way.

But it is necessary to note that between the transfer matrices used for static and dynamic calculations, there is some distinction.

So the transitional matrix of *i*-th section of the beam $\mathbf{U}_i$, used for dynamic calculations looks like [5]:

$$\mathbf{U}_i = \begin{bmatrix} A_i & \beta_i \cdot B_i & \dfrac{\beta_i^2 \cdot C_i}{\alpha_i} & \dfrac{\beta_i^3 \cdot D_i}{\alpha_i} \\ \dfrac{\lambda_i^4 \cdot D_i}{\beta_i} & A_i & \dfrac{\beta_i \cdot B_i}{\alpha_i} & \dfrac{\beta_i^2 \cdot C_i}{\alpha_i} \\ \dfrac{\alpha_i \cdot \lambda_i^4 \cdot C_i}{\beta_i^2} & \dfrac{\alpha_i \cdot \lambda_i^4 \cdot D_i}{\beta_i} & A_i & \beta_i \cdot B_i \\ \dfrac{\alpha_i \cdot \lambda_i^4 \cdot B_i}{\beta_i^3} & \dfrac{\alpha_i \cdot \lambda_i^4 \cdot C_i}{\beta_i^2} & \dfrac{\lambda_i^4 \cdot D_i}{\beta_i} & A_i \end{bmatrix}; \quad \alpha_i = E \cdot J_i / E \cdot J, \quad \beta_i = l_i / l, \quad \lambda_i^4 = m_i \cdot l_i^4 \cdot \omega^2 / E \cdot J_i, \quad (5)$$

where l_i, m_i, $E \cdot J_i$ – length, mass of length unit and bending stiffness of *i*-th area of the beam; l – general length of beam; $E \cdot J$ – bending stiffness of the beam section, appointed as main; ω –angular frequency of the transverse vibrations of the beam; A_i, B_i, C_i, D_i - hyperbolic functions.

In static calculations ($\omega = 0$) matrix $\mathbf{U}_i$ corresponds to the transfer matrix of stiffness of a beam section [5]:

$$\mathbf{U}_i = \begin{bmatrix} 1 & \beta_i & \dfrac{\beta_i^2}{2 \cdot \alpha_i} & \dfrac{\beta_i^3}{6 \cdot \alpha_i} \\ 0 & 1 & \dfrac{\beta_i}{\alpha_i} & \dfrac{\beta_i^2}{2 \cdot \alpha_i} \\ 0 & 0 & 1 & \beta_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (6)$$

Transfer matrix $\mathbf{G}_i$ is used only for dynamic calculations.

The form of transitional matrix $\mathbf{R}_i$ for static and dynamic calculations has almost identical form [5]:

$$\mathbf{R}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \sigma_i & 1 & 0 \\ -\varepsilon_i - \chi_i & 0 & 0 & 1 \end{bmatrix}; \quad \varepsilon_i = \frac{k_i \cdot l^3}{E \cdot J}; \quad \chi_i = j \cdot \frac{h_i \cdot \omega \cdot l^3}{E \cdot J}; \quad \sigma_i = \frac{k_{ni} \cdot l}{E \cdot J}, \quad (7)$$

where k_i, k_{ni} – radial and angular support stiffness on *i*-th beam section; h_i – a coefficient characterizing damping properties of supports (at $\omega = 0$ $\chi_i = 0$); $j = \sqrt{-1}$.

Obviously, that at the use of matrices (6) in equations (2) and (3), resulting transfer matrices of subsystems of tool $\mathbf{\Pi}^{(1)}$ and spindle $\mathbf{\Pi}^{(2)}$ will be upper triangular matrix. However, according to methodology [4], absence of non-zero elements below main diagonal at these matrices, does not allow to apply them for the calculation of influence coefficients.

On the other hand, using of matrices (6) in equation (4) in combination with the matrices (7) allows to get the acceptable type of resulting matrix $\mathbf{\Pi}^{(3)}$.

Thus, the method of decoupling of the «Spindle unit» system, accepted for dynamic calculations, can't be used for static calculations. But it ensues from the conducted analysis that such possibility appears when all subsystems are considered as the beams elastically mounted on the stationary foundation (like a subsystem 3, fig. 2). It can be provided by the decoupling of the system «Spindle unit» with the use of the mixed method of calculation of statically indefinable beam systems [9]. For this purpose in points the division of subsystems 2 and 3 to the subsystem 3 the reactions of throw-away connections ($-X_1, -X_2, -X_3, -X_4$) are being applied, and in a subsystem 2 additional connections are being introduced, with displacements ($q_2^{(3)}, q_3^{(3)}, q_4^{(3)}, q_5^{(3)}$) applied to them. In the same way, in points of subsystems 1 and 2 division, to the subsystem 2 the reactions of throw-away connections ($-X_0, -M_0$) are being applied, and in a subsystem 1 additional connections to that displacement ($q_0^{(2)}, \phi_0^{(2)}$) applied to them. Obtained chart of decoupling of the elastic system «Spindle unit» for static calculations is shown on a fig. 3.

Transfer matrices of subsystems $\mathbf{\Pi}^{(s)}$ for this chart of decoupling will be [4]:

for a tool ($s = 1$, $u = 2$): $\mathbf{\Pi}^{(1)} = \prod_{i=2}^{0} \mathbf{\Pi}_i^{(1)} = \mathbf{R}_2^{(1)} \cdot \mathbf{U}_2^{(1)} \cdot \mathbf{U}_1^{(1)}$; (8)

for a spindle ($s=2$, $u=6$): $\mathbf{\Pi}^{(2)}=\prod_{i=6}^{0}\mathbf{\Pi}_i^{(2)}=\mathbf{U}_6^{(2)}\cdot\mathbf{U}_5^{(2)}\cdot\mathbf{R}_4^{(2)}\cdot\mathbf{U}_4^{(2)}\cdot\mathbf{R}_3^{(2)}\cdot\mathbf{U}_3^{(2)}\cdot\mathbf{R}_2^{(2)}\cdot\mathbf{U}_2^{(2)}\cdot\mathbf{R}_1^{(2)}\cdot\mathbf{U}_1^{(2)}$; (9)

for the spindle unit housing ($s=3$, $u=7$): $\mathbf{\Pi}^{(3)}=\prod_{i=7}^{0}\mathbf{\Pi}_i^{(3)}=\mathbf{U}_7^{(3)}\cdot\mathbf{R}_6^{(3)}\cdot\mathbf{U}_6^{(3)}\cdot\mathbf{U}_5^{(3)}\cdot\mathbf{U}_4^{(3)}\cdot\mathbf{U}_3^{(3)}\cdot\mathbf{U}_2^{(3)}\cdot\mathbf{R}_1^{(3)}\cdot\mathbf{U}_1^{(3)}$, (10)

which means they meet the requirements [4] to the form of transfer matrices.

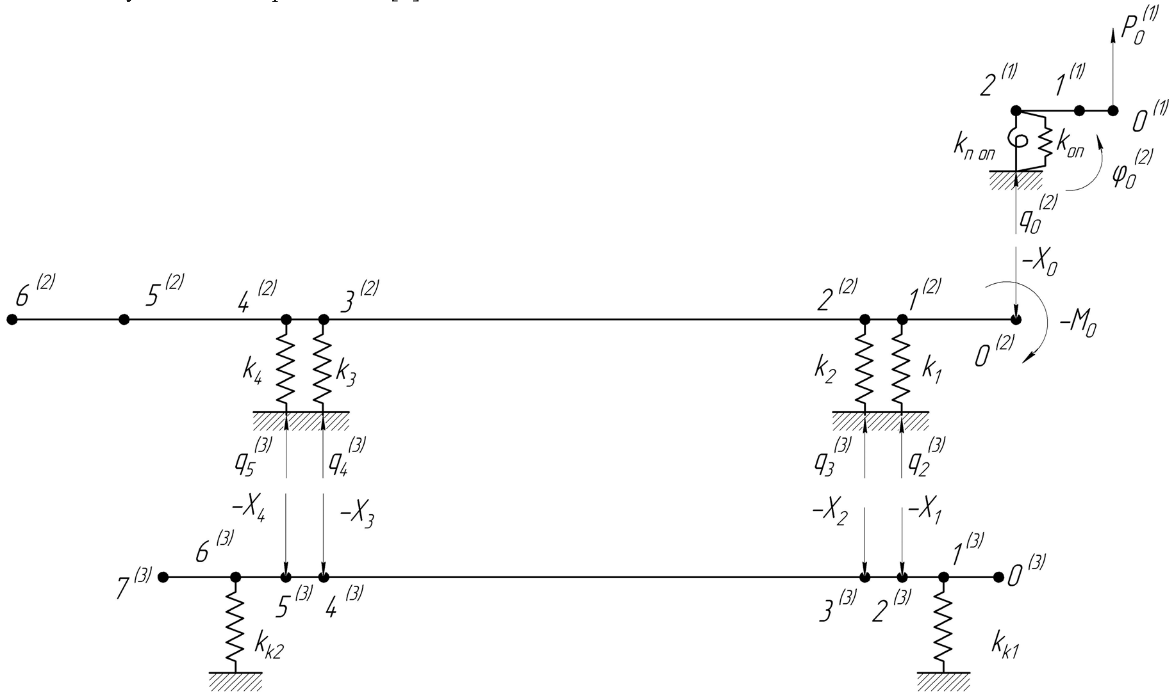

Fig. 3. Chart of decoupling of the elastic system «Spindle unit» for static calculations

Sequence of static calculation of the elastic system «Spindle unit»

In accordance with the chart of decoupling (fig. 3) for the tool subsystem ($s=1$) we will write down equations, for determination:

- displacements $q_2^{(1)}$ and $\phi_2^{(1)}$ in a point $2^{(1)}$:

$$q_2^{(1)}=\alpha_{20}^{(1)}\cdot P_0^{(1)}+q_0^{(2)} \text{ и } \phi_2^{(1)}=\beta_{20}^{(1)}\cdot P_0^{(1)}+\phi_0^{(2)}\,; \tag{11}$$

- reactions of additional connection X_0 and M_0 in a point $2^{(1)}$:

$$X_0=k_{on}\cdot(q_2^{(1)}-q_0^{(2)}) \text{ и } M_0=k_{n\,on}\cdot(\phi_2^{(1)}-\phi_0^{(2)})\,; \tag{12}$$

Considering (11), reactions of additional connection X_0 and M_0 in a point $2^{(1)}$ are determined by equations:

$$X_0=k_{on}\cdot\alpha_{20}^{(1)}\cdot P_0^{(1)} \text{ и } M_0=k_{n\,on}\cdot\beta_{20}^{(1)}\cdot P_0^{(1)}, \tag{13}$$

where $\alpha_{20}^{(1)}$ and $\beta_{20}^{(1)}$ – influence coefficients of the first subsystem: $\alpha_{20}^{(1)}$ - displacement and angle of turn in a point $2^{(1)}$ from the unit force, applied in a point $0^{(1)}$, $\beta_{20}^{(1)}$ - angle of turn in a point $2^{(1)}$ from the unit force, applied in a point $0^{(1)}$; k_{on} and $k_{n\,on}$ - radial and angular stiffness of the tool fixation in spindle; $P_0^{(1)}$ - external force, which is applied on a tool.

Spindle subsystem displacements ($s=2$) in points $q_1^{(2)},q_2^{(2)},q_3^{(2)},q_4^{(2)}$ of spindle supports (fig. 3), are determined by system of equations:

$$\begin{cases} q_1^{(2)}=-\alpha_{10}^{(2)}\cdot X_0-\gamma_{10}^{(2)}\cdot M_0+q_2^{(3)}+\delta_{12}^{(2)}\cdot q_3^{(3)}+\delta_{13}^{(2)}\cdot q_4^{(3)}+\delta_{14}^{(2)}\cdot q_5^{(3)} \\ q_2^{(2)}=-\alpha_{20}^{(2)}\cdot X_0-\gamma_{20}^{(2)}\cdot M_0+\delta_{21}^{(2)}\cdot q_2^{(3)}+q_3^{(3)}+\delta_{23}^{(2)}\cdot q_4^{(3)}+\delta_{24}^{(2)}\cdot q_5^{(3)} \\ q_3^{(2)}=-\alpha_{30}^{(2)}\cdot X_0-\gamma_{30}^{(2)}\cdot M_0+\delta_{31}^{(2)}\cdot q_2^{(3)}+\delta_{32}^{(2)}\cdot q_3^{(3)}+q_4^{(3)}+\delta_{34}^{(2)}\cdot q_5^{(3)} \\ q_4^{(2)}=-\alpha_{40}^{(2)}\cdot X_0-\gamma_{40}^{(2)}\cdot M_0+\delta_{41}^{(2)}\cdot q_2^{(3)}+\delta_{42}^{(2)}\cdot q_3^{(3)}+\delta_{43}^{(2)}\cdot q_4^{(3)}+q_5^{(3)} \end{cases}, \tag{14}$$

where $q_2^{(3)}, q_3^{(3)}, q_4^{(3)}, q_5^{(3)}$ – displacements of spindle unit housing subsystem ($s=3$)in points the division of second and third subsystems; $\alpha_{i0}^{(2)}$, $\gamma_{i0}^{(2)}$, $\delta_{ij}^{(2)}$ – influence coefficients of the second subsystem: $\alpha_{i0}^{(2)}$ - displacement in *i*-th points from the unit force, applied in a point $0^{(2)}$, $\gamma_{i0}^{(2)}$ - displacement in *i*-th points from the unit moment, applied in a point $0^{(2)}$, $\delta_{ij}^{(2)}$ - displacement in *i*-th points from the unit displacements, applied in *j*-th points.

Using dependences (13) we will distinguish components $q_{iP}^{(2)}$ in the equations of the system (14), that represents displacement in *i*-th points of the second subsystem from the action of the external load $P_0^{(1)}$:

$$q_{iP}^{(2)} = \alpha_{i0}^{(2)} \cdot X_0 + \gamma_{i0}^{(2)} \cdot M_0 = (k_{on} \cdot \alpha_{i0}^{(2)} \cdot \alpha_{20}^{(1)} + k_{n\,on} \cdot \gamma_{i0}^{(2)} \cdot \beta_{20}^{(1)}) \cdot P_0^{(1)} . \tag{15}$$

In addition, we will transform the influence coefficients $\delta_{ij}^{(2)}$ included in equations of the system (14). According to a theorem about reciprocity of reactions and displacement (to the 2th theorem of Rayleigh) [9]: $\delta_{ij} = -r_{ij}$, where δ_{ij} - are displacement in *i*-th points from the unit displacement, applied in *j*-th points, and r_{ij} - reactions in *i*-th points from the unit force, applied in *j*-th points. In turn, reactions in points the division of second and third subsystems (fig. 3): $r_{ij} = k_i \cdot \alpha_{ij}$, where α_{ij} - displacement in *i*-th points from the unit force, applied in *j*-th points, k_i - radial stiffness of supports, placed in *i*-th points ($i = 1,2,3,4$).On this basis, $\delta_{ij}^{(2)}$ in the system of equations (14) it is possible to write down:

$$\delta_{ij}^{(2)} = -k_i \cdot \alpha_{ij}^{(2)} . \tag{16}$$

Then considering (15) and (16) system of equations (14) will be:

$$\begin{cases} -q_1^{(2)} + q_2^{(3)} - k_1 \cdot \alpha_{12}^{(2)} \cdot q_3^{(3)} - k_1 \cdot \alpha_{13}^{(2)} \cdot q_4^{(3)} - k_1 \cdot \alpha_{14}^{(2)} \cdot q_5^{(3)} = q_{1P}^{(2)} \\ -q_2^{(2)} - k_2 \cdot \alpha_{21}^{(2)} \cdot q_2^{(3)} + q_3^{(3)} - k_2 \cdot \alpha_{23}^{(2)} \cdot q_4^{(3)} - k_2 \cdot \alpha_{24}^{(2)} \cdot q_5^{(3)} = q_{2P}^{(2)} \\ -q_3^{(2)} - k_3 \cdot \alpha_{31}^{(2)} \cdot q_2^{(3)} - k_3 \cdot \alpha_{32}^{(2)} \cdot q_3^{(3)} + q_4^{(3)} - k_3 \cdot \alpha_{34}^{(2)} \cdot q_5^{(3)} = q_{3P}^{(2)} \\ -q_4^{(2)} - k_4 \cdot \alpha_{41}^{(2)} \cdot q_2^{(3)} - k_4 \cdot \alpha_{42}^{(2)} \cdot q_3^{(3)} - k_4 \cdot \alpha_{43}^{(2)} \cdot q_4^{(3)} + q_5^{(3)} = q_{4P}^{(2)} \end{cases}, \tag{17}$$

Displacement $q_2^{(3)}, q_3^{(3)}, q_4^{(3)}, q_5^{(3)}$ included in the system of equations (17) of spindle unit housing subsystem ($s=3$) are determined by system of equations (fig. 3):

$$\begin{cases} q_2^{(3)} = -\alpha_{22}^{(3)} \cdot X_1 - \alpha_{23}^{(3)} \cdot X_2 - \alpha_{24}^{(3)} \cdot X_3 - \alpha_{25}^{(3)} \cdot X_5 \\ q_3^{(3)} = -\alpha_{32}^{(3)} \cdot X_1 - \alpha_{33}^{(3)} \cdot X_2 - \alpha_{34}^{(3)} \cdot X_3 - \alpha_{35}^{(3)} \cdot X_5 \\ q_4^{(3)} = -\alpha_{42}^{(3)} \cdot X_1 - \alpha_{43}^{(3)} \cdot X_2 - \alpha_{44}^{(3)} \cdot X_3 - \alpha_{45}^{(3)} \cdot X_5 \\ q_5^{(3)} = -\alpha_{52}^{(3)} \cdot X_1 - \alpha_{53}^{(3)} \cdot X_2 - \alpha_{54}^{(3)} \cdot X_3 - \alpha_{55}^{(3)} \cdot X_5 \end{cases}, \tag{18}$$

In turn, reactions X_1, X_2, X_3, X_4 (fig. 3) related to displacement of spindle and spindle unit housing subsystems in points of their division:

$$X_1 = k_1 \cdot (q_1^{(2)} - q_2^{(3)}),\ X_2 = k_2 \cdot (q_2^{(2)} - q_3^{(3)}),\ X_3 = k_3 \cdot (q_3^{(2)} - q_4^{(3)}),\ X_4 = k_4 \cdot (q_4^{(2)} - q_5^{(3)}) \tag{19}$$

Considering dependence (19) system of equations (18) is brought to a form:

$$\begin{cases} -k_1 \cdot \alpha_{22}^{(3)} \cdot q_1^{(2)} - k_2 \cdot \alpha_{23}^{(3)} \cdot q_2^{(2)} - k_3 \cdot \alpha_{24}^{(3)} \cdot q_3^{(2)} - k_4 \cdot \alpha_{25}^{(3)} \cdot q_4^{(2)} + \left(k_1 \cdot \alpha_{22}^{(3)} - 1\right) \cdot q_2^{(3)} + k_2 \cdot \alpha_{23}^{(3)} \cdot q_3^{(3)} + k_3 \cdot \alpha_{24}^{(3)} \cdot q_4^{(3)} + k_4 \cdot \alpha_{25}^{(3)} \cdot q_5^{(3)} = 0 \\ -k_1 \cdot \alpha_{32}^{(3)} \cdot q_1^{(2)} - k_2 \cdot \alpha_{33}^{(3)} \cdot q_2^{(2)} - k_3 \cdot \alpha_{34}^{(3)} \cdot q_3^{(2)} - k_4 \cdot \alpha_{35}^{(3)} \cdot q_4^{(2)} + k_1 \cdot \alpha_{32}^{(3)} \cdot q_2^{(3)} + \left(k_2 \cdot \alpha_{33}^{(3)} - 1\right) \cdot q_3^{(3)} + k_3 \cdot \alpha_{34}^{(3)} \cdot q_4^{(3)} + k_4 \cdot \alpha_{35}^{(3)} \cdot q_5^{(3)} = 0 \\ -k_1 \cdot \alpha_{42}^{(3)} \cdot q_1^{(2)} - k_2 \cdot \alpha_{43}^{(3)} \cdot q_2^{(2)} - k_3 \cdot \alpha_{44}^{(3)} \cdot q_3^{(2)} - k_4 \cdot \alpha_{45}^{(3)} \cdot q_4^{(2)} + k_1 \cdot \alpha_{42}^{(3)} \cdot q_2^{(3)} + k_2 \cdot \alpha_{43}^{(3)} \cdot q_3^{(3)} + \left(k_3 \cdot \alpha_{44}^{(3)} - 1\right) \cdot q_4^{(3)} + k_4 \cdot \alpha_{45}^{(3)} \cdot q_5^{(3)} = 0 \\ -k_1 \cdot \alpha_{52}^{(3)} \cdot q_1^{(2)} - k_2 \cdot \alpha_{53}^{(3)} \cdot q_2^{(2)} - k_3 \cdot \alpha_{54}^{(3)} \cdot q_3^{(2)} - k_4 \cdot \alpha_{55}^{(3)} \cdot q_4^{(2)} + k_1 \cdot \alpha_{52}^{(3)} \cdot q_2^{(3)} + k_2 \cdot \alpha_{53}^{(3)} \cdot q_3^{(3)} + k_3 \cdot \alpha_{54}^{(3)} \cdot q_4^{(3)} + \left(k_4 \cdot \alpha_{55}^{(3)} - 1\right) \cdot q_5^{(3)} = 0 \end{cases}, \tag{20}$$

By the joint decision of the systems of equations (17) and (20) displacements $q_1^{(2)}, q_2^{(2)}, q_3^{(2)}, q_4^{(2)}$ and $q_2^{(3)}, q_3^{(3)}, q_4^{(3)}, q_5^{(3)}$ are determined, and by dependences (19) - reactions X_1, X_2, X_3, X_4.

For determination of displacements $q_0^{(2)}$ and $\phi_0^{(2)}$, applied to the first from the side of second (fig. 3) one, we will replace an action got from (17) displacement $q_2^{(3)}, q_3^{(3)}, q_4^{(3)}, q_5^{(3)}$ by the equivalent action of corresponding reactions $X_{1q}, X_{2q}, X_{3q}, X_{4q}$ in spindle supports. According to (19) these reactions are determined by dependences:

$$X_{1q} = k_1 \cdot q_2^{(3)}, \quad X_{2q} = k_2 \cdot q_3^{(3)}, \quad X_{3q} = k_3 \cdot q_4^{(3)}, \quad X_{4q} = k_4 \cdot q_5^{(3)}, \tag{21}$$

The equivalent chart of subsystem of spindle got as a result of the conducted replacement is presented on a fig. 4, b. In accordance with this chart, displacements $q_0^{(2)}, \phi_0^{(2)}$ of the end of spindle will be determined by the system of equations:

$$\begin{cases} q_0^{(2)} = -\alpha_{00}^{(2)} \cdot X_0 - \gamma_{00}^{(2)} \cdot M_0 + \alpha_{10}^{(2)} \cdot X_{1q} + \alpha_{20}^{(2)} \cdot X_{2q} + \alpha_{30}^{(2)} \cdot X_{3q} + \alpha_{40}^{(2)} \cdot X_{4q} \\ \phi_0^{(2)} = -\beta_{00}^{(2)} \cdot X_0 - \phi_{00}^{(2)} \cdot M_0 + \beta_{10}^{(2)} \cdot X_{1q} + \beta_{20}^{(2)} \cdot X_{2q} + \beta_{30}^{(2)} \cdot X_{3q} + \beta_{40}^{(2)} \cdot X_{4q} \end{cases}, \tag{22}$$

or taking into account (15) and (21):

$$\begin{cases} -q_0^{(2)} + k_1 \cdot \alpha_{10}^{(2)} \cdot q_2^{(3)} + k_2 \cdot \alpha_{20}^{(2)} \cdot q_3^{(3)} + k_3 \cdot \alpha_{30}^{(2)} \cdot q_4^{(3)} + k_4 \cdot \alpha_{40}^{(2)} \cdot q_5^{(3)} = q_{0P}^{(2)} \\ -\phi_0^{(2)} + k_1 \cdot \beta_{10}^{(2)} \cdot q_2^{(3)} + k_2 \cdot \beta_{20}^{(2)} \cdot q_3^{(3)} + k_3 \cdot \beta_{30}^{(2)} \cdot q_4^{(3)} + k_4 \cdot \beta_{40}^{(2)} \cdot q_5^{(3)} = \phi_{0P}^{(2)} \end{cases}. \tag{23}$$

Taking into account displacements $q_0^{(2)}$ and $\phi_0^{(2)}$ (fig. 4,b), displacements of points $0^{(1)}$, $1^{(1)}$ and $2^{(1)}$ of a tool subsystem ($s=1$) are determined by dependences:

$$\begin{cases} q_{00}^{(1)} = q_0^{(2)} + \phi_0^{(2)} \cdot l_{02} + \alpha_{00}^{(1)} \cdot P_0^{(1)} \\ q_{10}^{(1)} = q_0^{(2)} + \phi_0^{(2)} \cdot l_{12} + \alpha_{10}^{(1)} \cdot P_0^{(1)} \\ q_{20}^{(1)} = q_0^{(2)} + \alpha_{20}^{(1)} \cdot P_0^{(1)} \end{cases}, \tag{24}$$

where l_{02}, l_{12} – are lengths of mandrel between sections 0-2 and 1-2 accordingly.

Thus, developed methodology allows to apply identical approach both for the dynamic and for static calculation of the elastic system «Spindle unit».

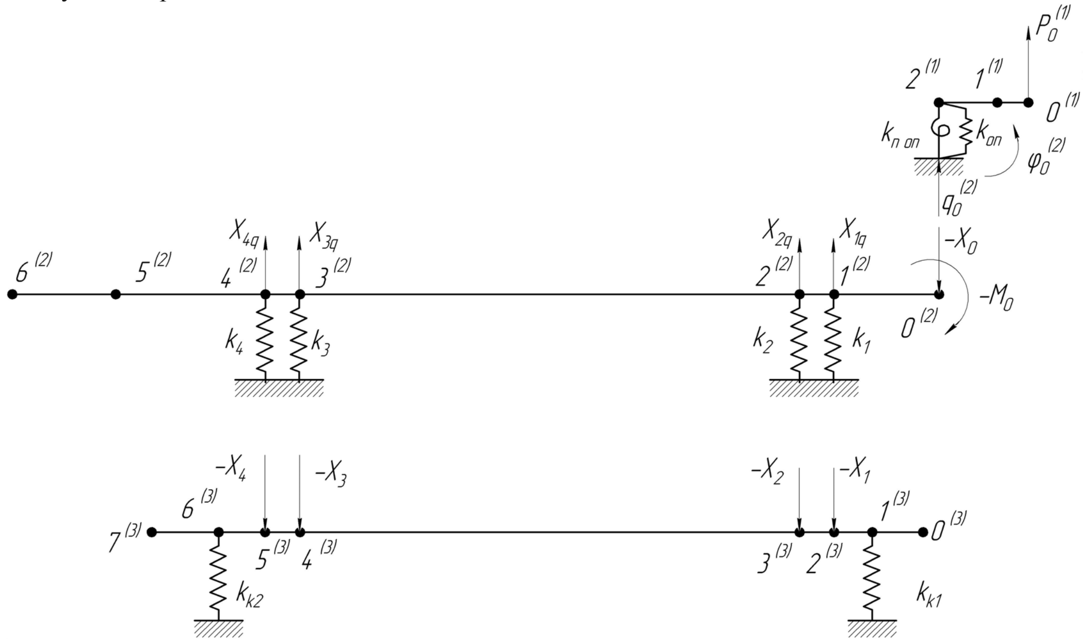

Fig. 4. Equivalent calculation charts of subsystems: a) tool; б) spindle; в) spindle unit housing

Conclusions

1. While using of transfer matrices method in the static and dynamic calculations of spindle unit as an elastic system, which consists of a few associate subsystems, it is necessary to apply the different methods of decoupling of the system. It is related to that during performing of decoupling with the use of method of dynamic compliances, transfer matrices of some subsystems in statics and dynamics have a different form. So, in statics these matrices have an upper triangular form, that excludes possibility of their use for the calculation of static compliance coefficients of subsystems, and, accordingly, performing of static calculation as such.

2. At static calculations for the decoupling of the system it is recommended to apply the combined method of calculation of statically indefinable beam systems. Application of this method allow to obtain the charts of subsystems, acceptable both to the calculation of static compliance coefficients of these subsystems and for the calculation of the elastic displacement in their characteristic points.

3. Offered approach allow to apply an identical methodological base for the simulation of both dynamic and static characteristics of the elastic system «Spindle unit».

Статичний розрахунок пружної системи «шпиндельний вузол» з використанням методу перехідних матриць

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Анотація. *Розроблена методика, яка дозволяє використовувати метод перехідних матриць для статичного розрахунку шпиндельного вузла, як пружної системи, що складається з декількох взаємопов'язаних підсистем. Сформульовані обмеження щодо використання методу динамічних податливостей для декомпозиції системи «шпиндельний вузол» при статичних розрахунках. Для таких обчислень запропоновано проводити декомпозицію системи з використанням змішаного методу розрахунку статично невизначених стержневих систем. Подана пружно-деформаційна модель системи «шпиндельний вузол», що складається з підсистем інструменту, шпинделя і корпуса, пружно закріпленого на станині верстату. Розроблена схема декомпозиції цієї системи при статичних розрахунках. Отримано аналітичний розв'язок задачі обчислення пружних переміщень в характерних точках підсистем. Розроблено алгоритм статичного розрахунку пружної системи «шпиндельний вузол». Запропонований підхід дозволяє при моделюванні статичних і динамічних характеристик пружної системи «шпиндельний вузол» застосовувати однакову методологічну базу.*

Ключові слова: *металорізальний верстат; шпиндельний вузол; пружно-деформаційна модель; декомпозиція пружної системи, метод перехідних матриць.*

Статический расчет упругой системы «шпиндельный узел» с использованием метода переходных матриц

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Аннотация. *Разработана методика, позволяющая использовать метод переходных матриц для статического расчета шпиндельного узла как упругой системы, состоящей из нескольких взаимосвязанных подсистем. Сформулированы ограничения использования метода динамических податливостей для декомпозиции системы «шпиндельный узел» при статических расчетах. Для таких расчетов предложено проводить декомпозицию системы с использованием смешанного метода расчета статически неопределимых стержневых систем. Представлена упруго-деформационная модель системы «шпиндельный узел», состоящей из подсистем инструмента, шпинделя и корпуса, упруго закрепленного на станине станка. Разработана схема декомпозиции этой системы при статических расчетах. Получено аналитическое решение задачи расчета упругих перемещений в характерных точках подсистем. Разработан алгоритм статического расчета упругой системы «шпиндельный узел». Предложенный подход позволяет при моделировании статических и динамических характеристик упругой системы «шпиндельный узел» применять одинаковую методологическую базу.*

Ключевые слова: *металлорежущий станок; шпиндельный узел; упруго-деформационная модель; декомпозиция упругой системы, метод переходных матриц.*

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