UDK 621.791.13:621.98.044

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MODELLING OF THE PROCESS OF ELASTIC-PLASTIC DEFORMATION OF A LAYERED METAL COMPOSITION WELDED BY EXPLOSION

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МОДЕЛИРОВАНИЕ ПРОЦЕССА УПРУГОПЛАСТИЧЕСКОГО ДЕФОРМИРОВАНИЯ СЛОИСТОЙ КОМПОЗИЦИИ, ПОЛУЧЕННОЙ СВАРКОЙ ВЗРЫВОМ

Purpose – to adapt the parametric mathematical model of the process of forming of a layered metal shell under pulse deformation based on the finite difference method, as well as to establish and compile the data on the mechanical properties and regularities of their distribution over the thickness of the layered work-piece.
Approach A bimetallic work-piece is covered with a spatial Lagrangian mesh X_1X_2 *connected with the middle surface. We use a "nodal scheme." The nodal point of the computational grid of each physical element is the point of bringing the corresponding mass equal to the sum of the masses of the bimetal components and applied to the center of gravity of the element. In the nodal points accelerations are determined. All other values are determined in each component. Mass component points are connected by weightless extensible direct links.*
Findings A methodology for calculation of technological parameters at the pulse deformation of layered compositions providing the plastic deformation of the contact surface layers was developed. On the basis of the finite-difference approximation of the equations modeling the dynamic behavior of the mechanics of a layered work-piece, algorithms for calculation of its kinematics and stress-strained state are proposed. The results of numerical calculations of the distribution of the radial deformation of the bimetallic stamped work-piece obtained by the explosive welding, change of deformation through the thickness of the bimetallic work-piece and distribution of the deformation intensity are shown.
Value Algorithms and programs for calculation of the dynamic behavior of a multi-layered shell, critical values of the stress intensities at which an interlayer destruction takes place are determined.
Keywords: layered metal composition; explosive welding; elastic-plastic deformation; two-layered shell; finite difference method.

The introduction. Currently, intensive researches aimed at the development and creation of advanced technologies related to providing products and manufacturing structures made of laminated metal are being carried out. A layered metal composition is a material composed of two or more layers of dissimilar metals, and having characteristics different from the source metals.

The layered metal composites are widely used in the construction of aircrafts, transport vehicles, electrical high-voltage equipment and others. This encourages a practical interest in the theory of the elastic-plastic deformation of laminated plates and shells, taking into account the inter-laminar stresses and transverse shear deformation.

Most of the calculation methods of the elastic-plastic deformation can be adapted to the modeling of the layered pieces shaping.

When using the method of the solution of the approximate equilibrium equations and the equations of plasticity [1] the deforming forces are determined for each layer of a layered work-piece. The dependence of the shear stresses on the coordinate, coinciding with the direction of external load, must take into account the nature of the coupling or interaction between layers. The control of non-uniformity of the thickness hardening of each layer of work-pieces welded by explosion is extremely difficult and only a qualitative analysis of the process with large assumptions is possible. The heterogeneity of a strain state of the individual layers of a multilayer work-piece is determined from the condition that the amount of the selected element [2] is constant.

The work of the deformation of each layer of the work-piece is determined separately by the energy method when determining the technological parameters of laminated pieces deformation and the results are summarized:

$$W_n = \sum_{k=1}^{m} W_k \qquad (1)$$

where m – the number of layers in the package; W_k – the work of the work-piece deformation.

DOI: http://dx.doi.org/10.20535/2305-9001.2016.77.68158

The energy balance equation for a layered work-piece the layers of which have different mechanical properties (H – hard or weakly deformed area) and (M – soft, deformed area) separated by surface S, where speed is continuous, would be:

$$\iint_{\sum M} \bar{\sigma}^n \vec{v} d\sum + \int_{\sum H} \bar{\sigma}^n \vec{v} d\sum + \frac{d}{dt}\left[0,5\iiint_{W_M} \rho_M v^2 dw + 0,5\iiint_{W_H} \rho_H \vec{v}^2 dw\right] =$$
$$= \iiint_{W_M} \sigma_{ik}\xi_{ik} dw + \iiint_{W_H} \sigma_{ik}\xi_{ik} dw + \iiint_{W_M} \rho_M \vec{F}\vec{v} dw + \iiint_{W_H} \rho_H \vec{F} v dw \quad (2)$$

where ΣM, ΣH – the part of surface areas M and H, enclosed in the volumes W_M and W_H; ρ_M, ρ_H – the density of layers M and H, σ_{ik} – the components of the stress tensor; v – the module of the velocity vector $\vec{\sigma}_n$, $\vec{F}$ – the external forces, surface and volume; ξ_{ik} – the rate of tensor component deformation. Power relations are valid for all segments of the layered work-piece. The integrals of the form $\iint_s \bar{\sigma}^n \vec{v} ds$ on the mating surface are reduced.

Description and prediction of the parameters of the blank forming, including multilayered ones, under impact loading is advisable using numerical methods [3], including the finite difference method which appeared the most suitable for a computer simulation [4-6] of the dynamics of the elastic-plastic medium.

A qualitative analysis of the plastic deformation under the impact loading of the laminated body is possible when solving the problem of the uniaxial plastic compression [7]. The method of approximate solution of the equilibrium equations for the elements of each layer was used. The inertial components are determined from the equality of the force pulse applied to given mass and the elemental energy required to change the speed of some infinitely small mass in deformation zone.

The dynamics of the multilayer shell with its pulse deformation is most expedient to be described by the finite-difference parametric model, whose algorithm for solving generalizes the difference scheme used for the calculation of isotropic shells of a finite length [1]. The difference equations were supplemented by the expressions for the grid functions on the contact surface of layers [8]. Using these relations, we can find the displacements and stresses in all the points and layers of the deformable work-piece.

In general, it should be noted that at present sufficiently reliable methods for selecting technologies and universal methods for determining the power parameters of the processes and patterns of layered metal compositions forming have not been developed.

The purpose of the work - to adapt the parametric mathematical model of the forming of a layered metal shell under pulse deformation based on the finite difference method, as well as to establish and compile the data on the mechanical properties and regularities of their distribution over the thickness of the layered work-piece.

Material and results of researches. A theoretical analysis of forming processes of a layered work-piece should be carried out taking into consideration the result of technological cycle of manufacturing and treatment of the layered materials is different properties of the constituents and the structural and mechanical inhomogeneity in the connection zone. This determines the specific behavior of the laminates at different shaping and calibration operations. The heterogeneity of the stress-strained state of layered compositions at loading with a deforming force determines the difference in the mechanical properties of the layers. At the initial stage the entire cross section of the layered composition works elastically. Then, in a layer with a lower elastic limit the stresses reach the yield stresses. Further deformation is determined by the supply of plasticity of this layer. The greatest danger is the area adjacent to the connection zone with the layer with a pronounced loss of strength.

The layers with a loss of strength emerging in the formation of the weld during the explosive welding influence the stress-strained state, which essentially depends on the stress pattern. In case of the difference of the elastic module of the base metal (Me), and the layer with a loss of strength (L) the stresses are typically defined as [9]

$$\sigma_{Me} = \sigma \frac{E_L}{hE_L + HE_{Me}}, \quad \sigma_L = \sigma \frac{E_{Me}}{hE_L + HE_{Me}} \quad (3)$$

where H and h – the relative thickness of the base metal layer and the layer with a loss of strength. When $E_L < E_{Me}$, the tension in the soft interlayer is lower than in the base metal. However, during the transition to the plastic area the states are possible when the tensile properties and the resource of plasticity of the weak link will be exhausted sooner than of the total composition. The layers with a loss of strength arising in the area of connections, welded by explosion, can be a source of damage. Deformations are concentrated in the softened layers, and the most solid areas are slightly deformed. When the deformation of the intermittent connections takes place the ratio of volumes and component properties play an important role at which there is a preferred deformation of the layers with a loss of strength with the exclusion of brittle layers. This allows a plastic deformation of connections with the wavy structure. The main indicator of the deformability of the bimetal is the ratio [9]

$$K_1 = \frac{\varepsilon_{\text{соед}}^{\Sigma} - \varepsilon_{\textit{комп}}^{\Sigma}}{\varepsilon_{\textit{комп}}^{\Sigma}}, \tag{4}$$

where $\varepsilon_{\textit{соед}}^{\Sigma}$, $\varepsilon_{\textit{комп}}^{\Sigma}$ – the average joint deformation of the connection area and composition.

When $K_1 \to 0$, there is a joint deformation of the connection zone and bimetal in general. The values of $K_1 << 0$ characterize the limited deformation capacity of the connection area. The plastic behavior of the metal is greatly influenced by the possibility of compensation of the low deformation ability of solid areas characterized by the coefficient

$$K_3 = \frac{\varepsilon_{\min}^{T} - \varepsilon_{\textit{комп}}^{\Sigma}}{\varepsilon_{\textit{комп}}^{\Sigma}}, \tag{5}$$

where $\varepsilon_{\min}^{T}$ – the local (average) deformation of solid structure components of the connection area.

High plasticity of soft zones ($K_2 >> 0$), where

$$K_2 = \frac{\varepsilon_{\max}^{M} - \varepsilon_{\textit{комп}}^{\Sigma}}{\varepsilon_{\textit{комп}}^{\Sigma}}, \tag{6}$$

where $\varepsilon_{\max}^{M}$ – the local (average) deformation of the soft components of the structure in the connection area which compensates for low deformation ability of the solid areas ($K_3 < 0$).

The evaluation of the ability to the plastic deformation of the bimetal welded by explosion is determined by the terms of deformation in its component joints and the condition of a quasi-joint deformation in the connection zone [9]:

$$\delta_{\Sigma}(M, C) \cong \delta_c \left(\Delta_{\max}^{M}, \Delta_{\min}^{T}\right), \tag{7}$$

where δ_{Σ}(M, C) – the average joint deformation of structural steel (M) and the cladding layer (C); $\delta_c(\Delta_{\max}^{M}, \Delta_{\min}^{T})$ – the average total deformation of the connection zone sections.

The possibility to engage stronger connection zones into the plastic deformation is characterized by the expression [9]

$$\sigma_T^H = \frac{\sigma_{0,2}^{T}}{\sqrt{3}K_s}\left(2 + \frac{1}{2\kappa}\right), \tag{8}$$

where σ_T^H – the average normal stress at the beginning of the plastic deformation of a solid component; κ – the coefficient of the contact hardening; K_s – the coefficient of mechanical heterogeneity, equal to

$$K_s = \frac{\sigma_{0,2}^{T}}{\sigma_{0,2}^{M}}, \tag{9}$$

where $\sigma_{0,2}^{M}$ и $\sigma_{0,2}^{T}$ – the yield stress of soft and hard components.

The connection of materials with different chemical composition and physic-mechanical properties leads to boundary defects due to a sharp mechanical heterogeneity, the emergence of structures of high hardness and low ductility, combined with soft, weakened areas in various combinations [9,10], which may be the potential sources of damage during forming.

Thermal effects due to the technological cycle of manufacturing products (normalization, tempering, heating for treatment under pressure) cause the redistribution of elements due to the thermodynamic instability at the interface.

Depending on the mode of the explosive welding, the line of connections may vary from a flat to undulating ordered or disordered character. The inclusions can take the form of a flat, bubble or indeterminate layer, including those with discontinuities of a shrinkage or gas origin [9]. The connecting layer of metals with the wavy structure after the explosive welding is jointing alternating sections with far different properties depending on the composition and modes of the explosive welding. Therefore, the deformability of different parts of the connection is different. The model of the plastic deformation of the layered work-piece can be significantly complicated by introducing several additional layers, which would take into account the heterogeneity of interlayer connection. However, the total thickness of the transition zone does not generally exceed 0.3–0.5 mm, and its mechanical properties are determined as mid-integral. With a layer thickness of 1.5–2.5 m, it is quite acceptable and justified. During the shear deformation characteristic of the bimetals the transition zone of a more durable material may not be taken into account. The strength of the connection area can be expressed as $\sigma_{в}^{con} = \sigma_{в}^{M} \cdot \kappa$, where **K** – the coefficient of strain- speed hardening.

When splitting each component of the layered work-piece into layers those layers which are in the interlayer interaction must have the mechanical properties differing from the inner ones by the magnitude of strain- speed hardening.

Thus, the determining factors for the description of the elastic-plastic deformation of the welded by explosion work-pieces will be the following:

a) the detonation of the explosive material change located on the plate being thrown which results in welding of work-pieces and a subsequent deformation of the work-pieces on the due;

b) the heat emission in the zone of collision of the welded work-pieces that improves the plastic properties and formability of the bimetal;

c) the change of high compressive stresses in the collision area by tensile stresses stimulates the process of deformation and the concomitant resonance phenomena;

d) the direct deforming of the work-piece.

Note that during the explosive welding the processes of the jet and wave formation and wave formation practically does not influence the deformation process and are not taken into consideration.

In the process of combined operations of the explosive welding and stamping the impact is completely plastic with the coefficient of recovery equal to zero, since the stresses are greater than twice the value of the yield strength of the deformed and welded materials. In the process of deformation at its final stage, the collision of a work-piece with the surface of the die, which is convenient to be set by a rigid undeformable contour, takes place.

It should be added that a short burst of the explosive charge can cause a loss of the stability of the deformable package and significantly change the forming character. The unloading, the impact of which is accompanied by the appearance of the opposing stretching waves leads to a strong local heating of the center of the deformable pieces and significant increase of their plastic properties.

When solving the equations of the dynamics of multi-layered work-pieces the d'Alembert principle and the Hamilton-Ostrohradskyi variation principle are used. The variation principle, which chooses a true motion from all possible motions and takes the system from the same initial position in the same period of time into the same end position, is characterized by the fact that the action integral takes the stationary value for it, i.e. $\delta J = 0$. The variation principle provides a reliable criterion of conservation of dynamic members. The dynamic components taken into consideration in the equations of dynamics should be conformed to the nature of the accepted hypotheses in the main part of the equation, i.e. with a type of the stress-strained state.

The deformation dynamics of layered work-pieces with greatly different plastic properties (hard and soft) is described considering the fact that hypothesis of Kirchhoff-Love is chosen for hard layers and for soft layers it is sufficient to consider only the transversal and lateral shear strain. When using work-pieces with similar mechanical properties it is necessary to consider the transverse shear and normal deformation. The model controlling the plastic deformation should be constructed at a sufficiently correct formulation of the plasticity criteria, selection of the appropriate theory of plasticity and deformation scheme. The absence of plastic deformations in some layers of a multilayered work-piece does not fully prevent the formability of the multilayered work-piece.

Thus, there is the problem of determining the limiting degree of deformation of multilayered work-pieces, to the solution of which there are different approaches. According to one of them the plastic deformations occur only in the layers of increased hardness. The material of soft layers remains resilient [11,12]. In another approach, the problem is solved under the assumption of a perfect elasticity of the material layers and the conditions of plasticity are tested. For those layers in which these conditions are violated, the relations which are the criterion of Mises [11,13] are accepted.

The resulting plastic deformation in the softer layers allows you to allocate a number of specific areas [10,13] for a contacting harder layer. The first – in both adjacent soft layers the plastic deformations are absent; the second – these layers have plastic deformations; the third – the plastic deformations are in the soft layer with number k; the fourth – the plastic deformations are in the soft layer with number k–1. It is also necessary to take into account the discontinuity of the fields of defining parameters in the description and modeling of the deformation process. There are gaps of density ρ (shaping layered pieces of dissimilar materials), speed (batch stamping) $\bar{\dot{x}}$, stress σ_{ik}. These gaps are subjected to certain conditions and integral relations, and their in-depth analysis was performed in [14-16].

During the deformation of a layered work-piece the kinematic conditions should be considered. On surface Σ(t) of the layer boundary there is the equality

$$\rho^{+}\left(\dot{x}_{n}^{+}-\psi\right)=\rho^{-}\left(\dot{x}_{n}^{-}-\psi\right), \tag{10}$$

where $\dot{x}_n^{+}$ – the normal velocity of the particles movement; ψ – the normal velocity of the surface of the layers boundary; "+", "–" the values of parameters on both sides of surface Σ; ρ^{+} and ρ^{-} – the density values.

The dynamic conditions obtained from the theorem of the momentum change have the form

$$\left[\sigma_{ik}\right]n_{k}=\rho^{+}\left(\dot{x}_{n}^{+}-g\right)\left[\dot{x}_{i}\right], \tag{11}$$

where $[\sigma_{ik}]$ и $\left[\dot{x}_i\right]$ – the surges of stresses and speeds when passing through surface Σ(+):

$$\begin{aligned}\left[\sigma_{ik}\right]&=\sigma_{ik}^{-}-\sigma_{ik}^{+},\\ \left[\dot{x}_{i}\right]&=\dot{x}_{i}^{-}-\dot{x}_{i}^{+}\end{aligned} \tag{12}$$

We will describe the energy conditions for a stock of homogeneous work-pieces deformation. The density and velocity have no discontinuities, at the same time surface tensions $\Sigma_k(t)$ (k = 1,2,...,n) are discontinuous. The conditions for the stresses have the form $[\sigma_{ik}]n_k$=0 or $\left(\vec{\sigma}^n\right)^+ + \left(\vec{\sigma}^n\right)^- = 0$. Adding the mechanical energy equations separately for each layer, we note that all the integrals $\iint_\Sigma \vec{\sigma}^n \vec{\dot{x}} d\Sigma$ on the discontinuity surface are reduced. Thus, the existence of discontinuity of stresses does not change the form of the equation of balance of works.

The stresses and densities are continuous on the discontinuity surface Σ_k (k = 1,2,...,n), but the vector of velocity has a discontinuity, so $\left[\vec{\dot{x}}\right] = \vec{\dot{x}}^- - \vec{\dot{x}}^+ \neq 0$. Having written the equation of the conservation of the mechanical energy for each layer in which the stresses, density and velocities are continuous, and added these equations for each surface Σ_k we obtain additional components of the form

$$\iint_{\Sigma_k} \left(\vec{\sigma}_n\right)^+ \left(\vec{\dot{x}}\right) d\Sigma = -\iint_{\Sigma_k} \tau_\nu \left[\vec{\dot{x}}\right] d\Sigma, \qquad (13)$$

where $\left[\dot{x}\right] = \left[\dot{x}^+ - \dot{x}^-\right]$ – the absolute value of the gap of the tangential velocity; τ_ν – the projection of the tangent component $\left(\vec{\sigma}^n\right)^+$ of the stress vector in the direction of the vector $\left[\vec{\dot{x}}\right]$.

The mechanical energy balance equation can be written as:

$$\iint_{\Sigma_M} \vec{\sigma}^n \vec{\dot{x}} d\sum + \int_{\Sigma_m} \vec{\sigma}^n \vec{\dot{x}} d\sum + \frac{d}{dt}\left[0,5\iiint_{W_m} \rho_m \vec{\dot{x}}^2 dw + 0,5\iiint_{W_M} \rho_M \vec{\dot{x}}^2 dw\right] = ,$$
$$= \iiint_{W_M} \sigma_{ik}\xi_{ik} dw + \iiint_{W_m} \sigma_{ik}\xi_{ik} dw + \iiint_{W_M} \rho_M \vec{F}\vec{\dot{x}} dw + \iiint_{W_m} \rho_m \vec{F} v dw. \qquad (14)$$

where w – the deformable area.

The components included in the layered metal compositions can have both the same and very different properties. When calculating the processes of the plastic deformation of layered pieces in some cases it is advisable to use the fundamental rheological models and their combinations [15, 16].

Comparison of mechanical properties of various materials, such as various grades of steel, titanium alloys, aluminum, copper, magnesium, etc., and manufactured from them bimetals enables to identify the most typical types of layered compositions.

The ratio of strength parameters of the layers and the degree of their hardening during the deformation characterizes the degree of their mechanical heterogeneity. On this basis, the welded by explosion compositions are divided into five main types: H-S-H, H-S-AH, H-S-HE, HE-S-HE, H-S-MH-H (H – hard, S – soft, AH – average hardness, HE – hard–elastic, MH – more hard).

In conditions of plastic deformation for bimetallic compositions (H-S, HE-S, MH-S, AH-S) it is necessary to have a soft component. A schematization of the rheological models of elastic-viscous-plastic compositions is performed by the series-parallel connections, combining elastic, viscous, plastic and destructible elements. A SE (soft-elastic) element is likely to be added to the types being used.

Let us consider the case of a two-layered metal composition (hereinafter - bimetal). A bimetallic work-piece is covered with a spatial Lagrangian mesh X_1X_2 connected with the middle surface. We use a "nodal scheme." The nodal point of the computational grid of each physical element is the point of bringing the corresponding mass equal to the sum of the masses of the bimetal components and applied to the center of gravity of the element. In the nodal points accelerations are determined. All other values are determined in each component. Mass component points are connected by weightless extensible direct links.

We split each layer of a bimetallic work-piece into sub-layers, four in each layer. These layers are arranged equidistant from one another, operate in a planar stressed state and are separated by the material which cannot operate under similar conditions. In this model, the hypotheses that give an assumption of maintaining the normal element geometry are valid for thin work-pieces. For work-pieces of a medium and large thickness the hypotheses are replaced by the assumptions based on cross- shear deformations. We believe the materials of the layers of a bimetallic work-piece to be isotropic elastic-plastic with the hardening, the value of which depends on the coordinate along the symmetry axis of the work-piece.

We determine the acceleration, velocity and displacement from the equilibrium equations for each mn-th node of the work-piece

$$\begin{cases} \nabla_\gamma M_{mn}^{\beta\alpha} - Q_{mn}^{\beta} R_{\gamma mn}^{\beta} + P_{mn}^{\alpha} + T_{mn}^{\alpha} + S_{mn}^{\alpha} = \bar{\rho}\ddot{X}_{mn}^{\alpha} - \rho\dot{X}_{mn}^{\alpha} c, \\ M_{mn}^{\beta\alpha} R_{\beta\alpha}^{mn} + \nabla_\beta Q_\beta^{mn} + P_{mn}^{3} + T_{mn}^{3} + S_{mn}^{3} = \rho\ddot{X}_{mn}^{3} - \rho\dot{X}_{mn}^{3} c, , \\ \nabla_\beta L^{\alpha\beta} - Q_{mn}^{\alpha} = 0 \end{cases} \qquad (15)$$

where ∇_β – a sign of covariant differentiation; M_{mn} – the membrane forces; Q^β_{mn} – the shear forces; R_{mn} – the curvature tensor; P^j_{mn} – the force of the impulse load; T_{mn} – the friction force in the peripheral area of the work-piece; S_{mn} – the forces of the hindering elements of the die; $\bar{\rho}$ – the reduced mass; $\ddot{X}^j_{mn}$ – the acceleration; c – the speed of sound in the subsequent work-piece; L – the bending moments.

We replace the system (15) of differential equations by the finite-difference analogue. In the model the sub-layers constituting each layer are separated from each other by the material with shear modulus G^i and plastic shear resistance τ_s^i. The whole bending is concentrated at the locations of the masses.

The calculation is performed in the following order. The first step is to split a multi-layer shell by elements and sub-layers, to determine the initial and boundary conditions. The rheological material model is selected due to its physical properties. The components of acceleration $\ddot{x}_1$ and $\ddot{x}_2$ in all units and positions of the nodes at the initial time $\tau \to 1$ are determined from the expressions that define the acceleration:

$$x^i_{1,\tau+1} = \ddot{x}^i_{1,\tau+1} \cdot \Delta t^2 + 2x^i_{1,\tau} - x^i_{1,\tau-1}, \quad (16)$$

$$x^i_{2,\tau+1} = \ddot{x}^i_{2,\tau+1} \cdot \Delta t^2 + 2x^i_{2,\tau} - x^i_{2,\tau-1}, \quad (17)$$

where Δt – the integration step over time size; τ – the number of the time step.

To determine the plastic strain increment we use the adjusted incremental theory of plastic flow for each sub-layer. The corresponding stresses in the layers and sub–layers are obtained from the approximate (finite-difference) equations for stresses and strains. We determine the membrane (axial) forces and moments at the locations of mass points [16] by stresses. The moment equation (15) allows calculating the shear forces. Knowing the coordinates of the reduction points and taking into account the conditions of the volume constancy, we calculate the amounts of relative deformations in sub-layers of each layer. After this we find the strain increments $\Delta\varepsilon^i_{43,\tau+1}$, $\Delta\varepsilon^i_{4,\tau+1}$ and $\Delta\varepsilon^i_{3,\tau+1}$

Assuming that the change in stress is in accordance with the generalized Hooke's law, we determine the increment of stresses. Wherein each layer has elastic modulus E^i, shear modulus G^i and Poisson's ratio ν^i, where i is the number of layer:

$$\Delta\sigma_{3,\tau+1} = \frac{E^i}{1-\nu_i^2}(\Delta\varepsilon^i_3 + \nu^i \Delta\varepsilon^i_4)_{\tau+1}, \quad (18)$$

$$\Delta\sigma_{4,\tau+1} = \frac{E^i}{1-\nu_i^2}(\Delta\varepsilon^i_4 + \nu^i \Delta\varepsilon^i_3)_{\tau+1}, \quad (19)$$

$$\Delta\sigma_{34,\tau+1} = G^i \Delta\varepsilon_{34}, \quad (20)$$

where ν^i – the Poisson's ratio of the i–th layer, G^i – the shear modulus of the i–th layer.

Then, a check if the resulting stresses are within the area bounded by the flow curve is carried out, i.e. the generalized criterion of Huber-Mises yield $F_{\tau+1}$ is calculated.

At the boundary of the layers the conditions of continuity of the strain and stresses are carried out. When determining the accelerations, the work-piece is reduced to a single-layer work-piece, the other parameters are determined for each component (Fig. 1).

Then the cycle is repeated with the determination of the acceleration of the grid points of the middle surface of the laminar work-piece on the next stage of integration according to dependence (15)

$$\ddot{x}^j_{mn} = \frac{A^{0,5}_{mn}}{\bar{\rho}_o} \cdot \left(P^j_{mn} + T^j_{mn} + S^j_{mn} + \Pi^j_{mn} + \left(\frac{\partial V^{\beta j}_{mn}}{\partial x^\beta} + \bar{A}^\beta_{mn} \cdot \frac{\partial \bar{A}^{mn}_\gamma}{\partial x^\beta} \cdot V^{\gamma j} \right) \right), \quad (21)$$

where A_{mn} – the determinant of the metric tensor, $V^{\alpha j}_{mn}$ – the space-surface tensor.

Timeframe Δt is chosen from the condition of stability of the calculating process [11]

$$\Delta t \le \Delta X_{5,j,0} \left[\rho^k_3 (1-\nu^k)^2 (E^k)^{-1} \right]^{0,5}, \quad (22)$$

where ρ^k_3 – the density of the k-th material of the work-piece; ν^k and E^k – the Poisson's ratio and Young's modulus of the k-th material of the work-piece, respectively.

According to the proposed algorithm the program "MALTL" has been developed, allowing to carry out the calculation of the stress-strained state in the process of the forming of a layered shell and determine the final deflections of a round layered work-piece subjected to the explosive loading.

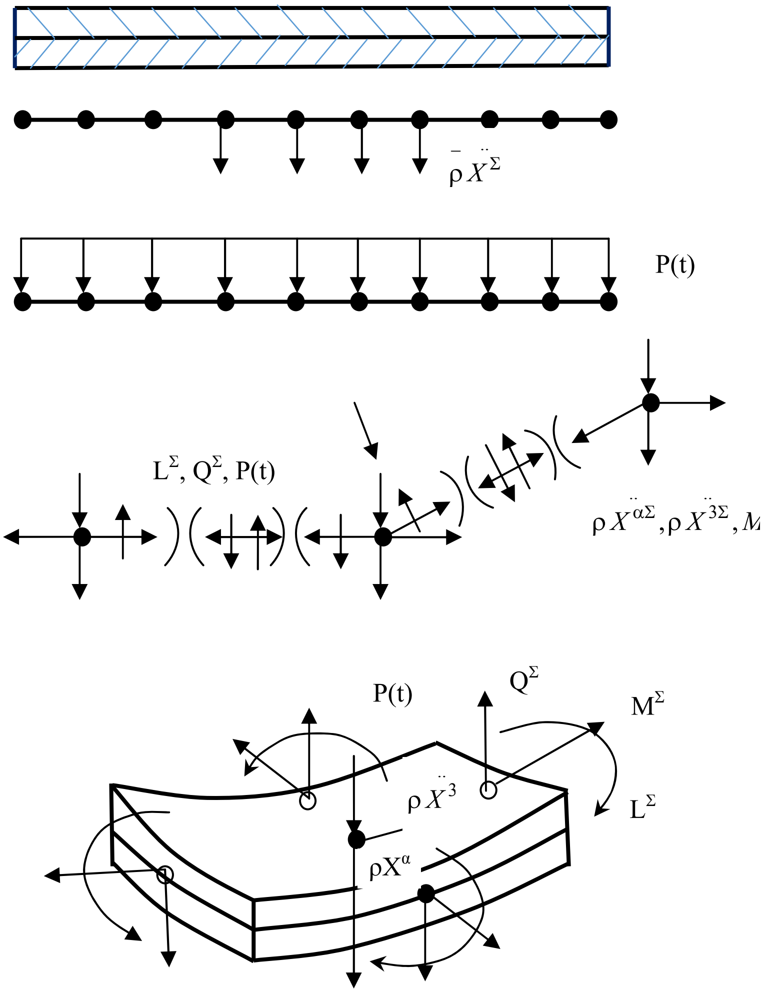

Fig. 1. Calculation scheme of elastic-plastic deformation of the two-layered shell

In the modeling of the pulse deformation process the data were input for a layered work-piece (steel 12X18H10T – aluminum alloy AMr2M) that are given in tables 1 and 2.

The diameter of the work-piece is D_s = 0.2 m, the thickness of the aluminum alloy layer is δ_1 = 1.5 mm, the thickness of the steel layer is δ_2 = 3 mm.

Table 1

Mechanical properties of layered work-piece materials obtained by the explosive welding used for the explosive forming

Material	Mechanical properties					
	$E\cdot10^5$, МПа	E_p, МПа	σ_s, MPa	$\sigma_в$, MPa	ε_b	ε_p
12X18H10T	2,06	739	299	589	0,45	0,38
AMr2M	0,71	724	96	206	0,15	0,15

Table 2

Parameters of hardening of layered work-piece materials obtained by the explosive welding used for explosive forming

Material	Parameters of deformation hardening			Parameters of kinematic hardening	
	ε_s	λ	a*	D, c^{-1}	χ
12X18H10T	1,43	0,992	0,124	396	7,14
AMr2M	1,13	0,996	0,15	$5{,}62\cdot10^6$	3,75

The results of measuring the depth of some layers subjected to hardening are shown in Fig. 2.

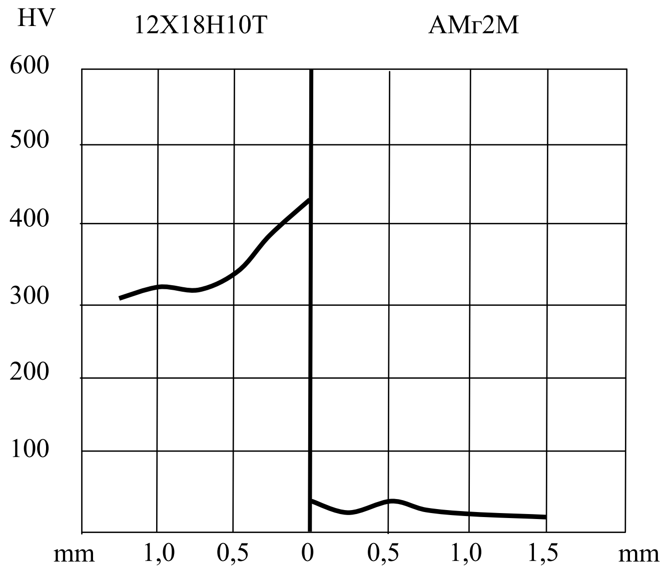

Fig. 2. Hardness in bimetal 12X18H10T + AMг2M connection area

The depth of the hardened layer reaches 0.4...0.6 mm, so the mechanical characteristics of the contact layers, the last for the cladding layer and the first for the cladded layer, are accepted increased.

Fig. 3 shows a graph of the displacement of the central point of the bimetallic work-piece produced by the explosive welding, and fig. 4 – when combining the operations of the explosive welding and forming.

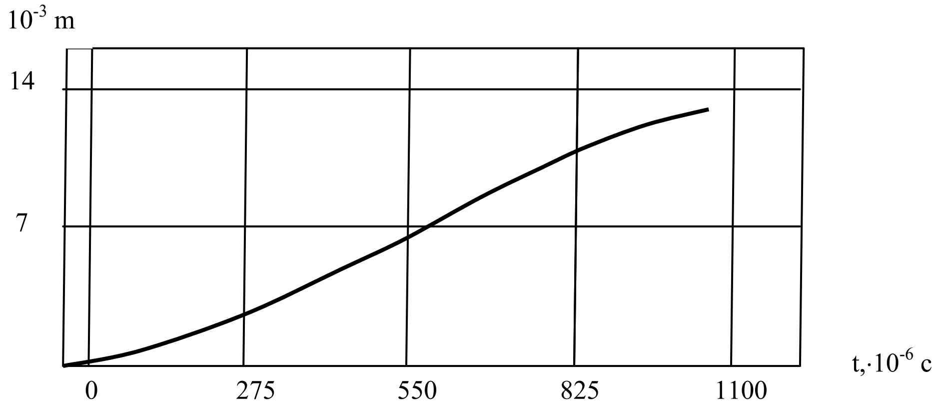

Fig. 3. Displacement of the center point of the bimetallic work-piece

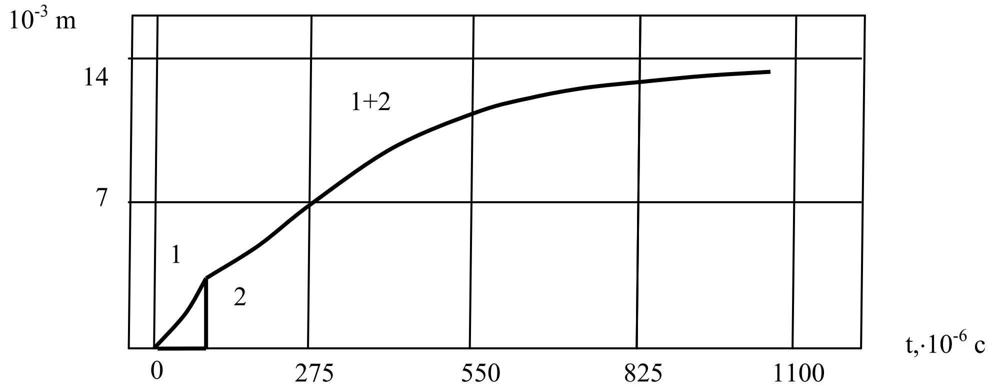

Fig. 4. Displacement of the center point of the work-piece by combining the operations of the explosive welding and forming: 1 – AMг2M; 2 – 12X18H10T

The results of numerical calculations of the distribution of the radial deformation of the bimetallic stamped work-piece obtained by the explosive welding are shown in fig. 5 and 6.

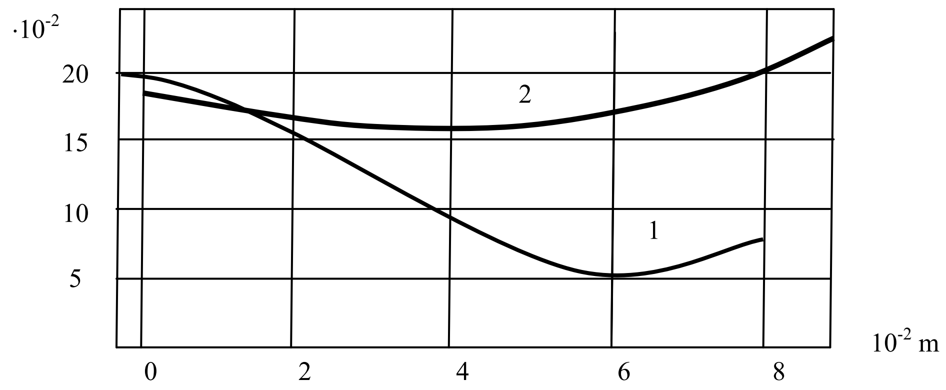

Fig. 5. Change of deformation through the thickness of the bimetallic work-piece:
1 – the explosive forming of the bimetal work-piece; 2 – the combination of the explosive welding and forming operations

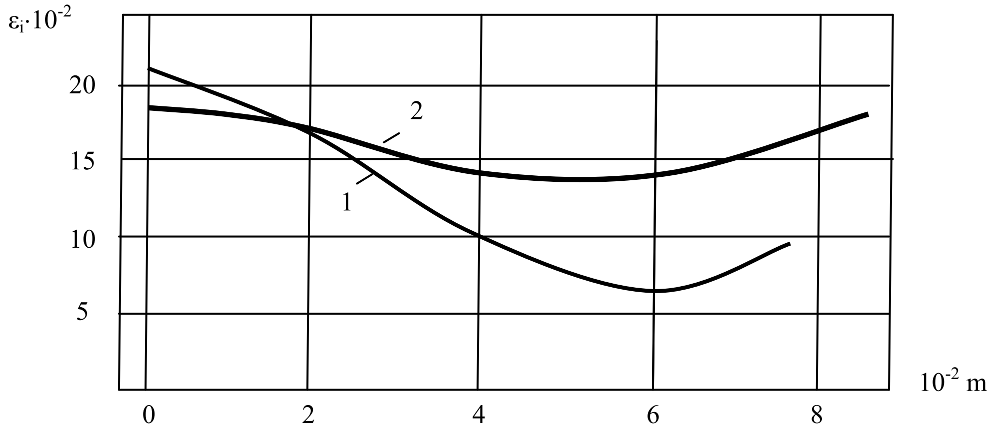

Fig. 6. The distribution of intensity of deformations: 1 – the explosive forming of the bimetal work-piece;
2 – the combination of the explosive welding and forming operations

Comparing the mathematical model describing the quantitative characteristics (displacement, velocity and acceleration) of the bimetallic deformable work-piece during deformation and in its final state, with those given in [17] experimental results shows that its error is within the confidence intervals.

Conclusions. 1. On the basis of the physical equations for the layered media a methodology for calculation of the technological parameters providing the plastic deformation of the contact surface layers at the pulse deformation of layered compositions was developed which allows doing calculations of the technological processes of different options of producing, shaping and calibration of layered pieces.

2. On the basis of finite-difference approximation of the equations modeling the dynamic behavior of the mechanics of a layered work-piece, algorithms for calculation of its kinematics and stress-strained state are proposed.

3. In accordance with the mathematical model of the dynamic behavior of a multi-layered shell and developed algorithms and programs for their calculation, critical values of the intensities of strain and stresses at which an interlayer destruction takes place are determined.

Анотація. *У статті аналізуються особливості методів розрахунку процесу деформування шаруватих металевих композицій. Дана оцінка неоднорідності структурно-механічного характеру, що виникають в процесі виробництва шаруватих металевих композицій, як в області з'єднання й товщини кожного шару. Розроблено математичну модель пружно-пластичного деформування з урахуванням кінематичного й деформаційного зміцнення композиційних матеріалів і особливостей будови і властивостей зони з'єднання. Математична модель враховує неоднорідність механічних і фізичних властивостей перехідної зони шляхом введення додаткових підшарів. Представлені кінематичні й динамічні граничні умови по шарам. Наведено графічні залежності інтенсивності деформації від товщини заготовки і переміщення центральної точки шаруватого композиту. Надано розрахунок для композиції нержавіюча сталь-алюмінієвий сплав.*
Ключові слова: *багатошарова композиція металу; зварювання вибухом; пружно-пластична деформація; двошаровий оболонка; метод кінцевих різниць.*

Аннотация. *В статье анализируются особенности методов расчета процесса деформирования слоистых металлических композиций. Дана оценка неоднородности структурно-механического характера, возникающие в процессе производства слоистых металлических композиций, как в области соединения и толщины каждого слоя. Разработана математическая модель упруго-пластического деформирования с учетом кинематического и деформационного упрочнения композиционных материалов и особенностей строения и свойств зоны соединения. Математическая модель учитывает неоднородность механических и физических свойств переходной зоны путем введения дополнительных подслоев. Представлены кинематические и динамические граничные условия по слоям. Приведены графические зависимости интенсивности деформации от толщины заготовки и перемещения центральной точки слоистого композита. Дан расчет для композиции нержавеющая сталь-алюминиевый сплав.*
Ключевые слова: *многослойная композиция металла; сварка взрывом; упруго-пластическая деформация; двухслойная оболочка; метод конечных разностей.*

References

1. Mayboroda, V.P., Kravchuk, A.S. and Homin N.N. (1986), *Skorostnoe deformirovanie konstruksionyh materialov* [High-speed deformation of structural materials], Engineering, Moscow, Russia.
2. Mikhailuta, I.O. (1979), Analiz izmeneniy tolshini otdelnix sloev zagotovki pri paketnoy besprigimnoy shtampovke [Analysis of changes in the thickness of the individual layers of a workpiece during a stack stamping without a blank holder], Mikhailuta I.O., Poltarushnikov S.A., Volovik A.A., et al. (ed.) V.K. Borisevich, *Interuniversity thematic coll. of works scientific Pulse metal forming, N.Yr. Zhukovsky Kharkiv Aviation Institute named*, KH. KCAI, no 8, pp. 43-46.
3. Borisevich, V.K. (ed.) (1977), Interuniversity thematic coll. of works scientific Pulse metal forming, *Interuniversity thematic coll. of works scientific NN.Yr. Zhukovsky Kharkiv Aviation Institute named,* KH. KCAI, no 6, 144 p.
4. Doroshko, V.I. and Dubasov, V.M. (2001), *Vidavlivanie, imetallicheskix rabochih detalei shtampov I press-form* [Extrusion of bimetallic working parts of dies and moulds], Publisher VNU, Lugansk, Ukraine.
5. Kuzmenko, V.I. and Balakin, V.F. (1990), *Reshenie na EBM zadach plasticheskogo deformirovaniya* [Computer solution of plastic deformation problems], Tehnika, Kiev, Ukraine.
6. Vdovin, S.I. (1988), *Metody rascheta I proektirovaniya na EBM processov shtampovki listovih I profilnih zagotovok* [Computer methods of calculation and design of stamping sheet and profile pieces], Engineering, Moscow, Russia.
7. Dragobetsky, V.V. (2002), Reshenie odnomernoj zadachi plasticheskogo deformirovanija bimetallicheskih zagotovok pri impul'snom nagruzhenii [Solution of the one-dimensional problem of plastic deformation of bimetallic blanks under impact loading], *Metallurgical and Mining Industry,* no 8-9, pp. 507-511.
8. Guz, A.N., Kubenko, V.D. and Babayev, A.E. (1984), *Gidrouprugost' sistem obolochek* [Hydroelasticity of shell systems], Higber School, Kiev, Ukraine.
9. Konan, Yu.A. (1987), *Svarka vzryvom* [Explosion welding], Engineering, Moscow, Russia.
10. Belyaev, V.I., Pervukhin, L.B. and Chudnovsky, A.D. (1976), Vysokoskorostnaja deformacija metallov [High-speed deformation of metals], V.I. Belyaev, V.N. Kovalevsky, G.V. Smirnov, Science and technology, Minsk, Belorussija.
11. Bolotin, V.V. and Novikov, Yu.N. (1980), *Mehanika mnogoslojnyh konstrukcij* [Mechanics of multilayer constructions], Engineering, Moscow, Russia.
12. Vasidzu, K. (1987), *Variacionnye metody v teorii uprugosti i plastichnosti* [Variational methods in the theory of elasticity and plasticity], Translated by Kobelev, V., Seyranyan, A.P., in Banichuk, N.V. (ed.), Mir, Moscow, Russia.
13. Korolev, V.I. (1970), *Uprugo-plasticheskie deformacii obolochek* [Elastic-plastic deformations of shells], Engineering, Moscow, Russia.
14. Gulyaev, Y.G., Chukmasov, S.A. and Guba, A.V. (1986), *Matematicheskoe modelirovanie processov obrabotki metallov davleniem* [Mathematical modeling of metal forming processes], Naukova Dumka, Kiev, Ukraine.
15. Kolmogorov, V.A. (1986), *Obrabotka metallov davleniem* [Metal Forming], Metallurgy, Moscow, Russia.
16. Gong, G.Y. (1980), *Teoreticheskie osnovy obrabotki metallov davleniem* (Teorija plastichnosti) [Theoretical Foundations of metal forming (theory of plasticity)], Metallurgy, Moscow, Russia.
17. Dragobetsky, V.V. (2003), *Rozrahunok ta eksperimental'na perevirka napruzhenogo stanu sharuvatih materialiv pri impul'snomu navantazhenni* [Calculation and experimental test of the stressed state of layered materials during impulse loading], *News Kremenchutskogo sovereign politehnichnogo universitetu,* Naukovi pratsi KDPU, Kremenchuk, no 2 (19), pp. 82-84.

Подана до редакції